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A STUDY OF THE FACTORS AFFECTING THE RANGE OF AIRPLANES

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SUMMARY

A study has been made of the most important factors affecting the range of airplanes. In the first of three parts of the paper the various factors are individually analyzed and evaluated relative to each other in order to establish a basis for compromise in design. In the second part the effect of varying a number of the most important factors is determined for a sample airplane. In the third part the problem of take-off is investigated for the most critical design conditions encountered in part II and means for improving the take-off are analyzed.

The study, which is based upon certain reasonable assumptions, indicates the following generalities:

- (1) Changing the propeller pitch during flight was found to be of little value except for take-off and climb.
- (2) It was found desirable to design the propeller to absorb the power at a high value of engine torque in order that the fuel consumption might remain low.
- (3) A large span is desirable for obtaining the maximum range at low speeds; but less so for flight at high speeds.
- (4) It was found desirable, in certain instances, to sacrifice aerodynamic cleanness in order to reduce weight because of the overshadowing importance of weight for long-range airplanes.
- (5) Flight at either constant speed or constant power was found preferable to flight at constant L/D , assuming certain engine fuel consumption characteristics and provided that the average speed of flight is the same for all conditions.
- (6) A gain in speed without sacrificing range may be realized for any cruising value of angle of attack if the full-throttle engine power is reduced the desired amount by flying in the rarefied air at altitudes instead of by throttling the engine at the carburetor at a lower altitude.
- (7) There was no advantage discovered in increasing the size of unsupercharged engines in order to fly at high altitudes unless it became necessary to improve the take-off thereby.
- (8) It appeared, however, that a large gain in speed with little loss in maximum range could be obtained by supercharging to high altitudes, even

though the relatively inefficient gear-driven centrifugal supercharger was assumed. (9) It was found highly desirable to increase the design span and parasite loadings by increasing the gross weight for a given geometric size to the highest possible value consistent with take-off limitations. (10) Wing flaps were found to be effective in reducing the take-off run of heavily loaded airplanes provided that a certain minimum power was available, thereby making possible take-offs with greater loads.

INTRODUCTION

The range of an airplane is not often of primary importance in design because the length of flight is generally limited by other considerations. For commercial airplanes the points at which stops are made are usually well within the ordinary operating radius. But as air lines are established over wide areas of land or great stretches of water, the range of airplanes takes on a new importance.

The basic factors involved in the subject of range are generally understood but the relative importance of the numerous variables entering into these factors are less familiar. An examination of any of the various existing range formulas reveals that maximum range is dependent only on a simple relationship between the aerodynamic efficiency of the airplane and propeller; the thermal, mechanical, and weight efficiencies of the power plant; and the structural or weight efficiency of the airplane. To build an airplane of maximum possible range, one need only to build into a machine the characteristics giving the highest possible combination of these efficiencies with the peaks occurring at one condition of flight. A further study of the subject reveals that this condition of flight for maximum range is necessarily at a relatively low speed and altitude and that the take-off and climb characteristics of such an airplane are very poor.

In order to increase the utility of an airplane designed only for the maximum possible range it becomes necessary to compromise on some of the design features. It is obviously desirable to increase the speed; this increase may be accomplished by a number of methods, some of which require a considerable sacrifice of range. For most purposes, there is usually an allowable minimum limit for the take-off run and rate of climb. These characteristics may likewise be improved in a number of ways.

An attempt has been made in this study to analyze coordinately the important factors affecting range; to evaluate each factor, when possible, in terms of the others, thereby establishing a basis for compromise in design; to study various methods of improving the performance with their resulting effects on the range; to furnish data and methods for computations that might be useful in design work; and to determine, if possible, avenues for future research on the problem of range.

The first part of the paper is a discussion of the basic factors affecting range; the second gives numerical examples showing the effects of different variables on the range of a two-engine airplane; and the third part is an analysis of the take-off problem of long-range airplanes.

I - THE BASIC FACTORS AFFECTING RANGE

Range Formulas

The basis for most of the formulas commonly used for the determination of the range of airplanes may be expressed by the relation:

$$\text{Range} = \int \frac{\text{miles}}{\text{pound of fuel}} d(\text{fuel weight})$$

which takes the form, assuming flight at constant L/D

$$R = \frac{375\eta}{c} \frac{L}{D} \int_{W_e}^{W_o} \frac{dW}{W}$$

where R is the range in miles

η , propulsive efficiency.

c , mean specific fuel consumption, lb./b.hp.-hr.

$\frac{L}{D}$, lift-drag ratio of the airplane at angle of flight

W_o , weight of airplane at start of flight

and W_e , weight of airplane at finish of flight

Upon integrating, the expression assumes the form of the familiar Breguet formula,

$$R = 863 \frac{\eta}{c} \frac{L}{D} \log_{10} \left(\frac{W_0}{W_g} \right)$$

The greatest source of error in the use of this formula is the specific fuel consumption, which varies widely for different engines and for the same engine at different engine-control settings. In order to eliminate the error, Diehl (reference 1) has devised a method whereby reasonably accurate estimations of the specific fuel consumption can be made and has further modified the formula to give improved results. Other modifications have been made for the purpose of covering different flight conditions (reference 2). The most accurate and also the most flexible method is the step-by-step graphical integration method, wherein instantaneous values of miles/pound of fuel are plotted against fuel weight and integrated graphically. This method in an extended form was used in the computations made in this analysis and will be more fully discussed later.

Aerodynamic Considerations

The lift-drag ratio of the airplane.— From the standpoint of range alone the L/D is the only aerodynamic consideration of importance, aside from the propeller characteristics. Flights that have for their purpose the attainment of maximum range will be at speeds for which the mileage per pound of fuel is the greatest. Ordinarily these speeds will nearly coincide with the speeds for $(L/D)_{\max}$. Since flight at constant L/D also represents constant angle of attack, the true speed will vary with the weight and altitude thus:

$$\frac{V}{V_0} = \left(\frac{W}{W_0} \frac{\rho_0}{\rho} \right)^{1/2}$$

where V is the true air speed

W , the weight of the airplane at any instant

ρ , the density of the air

and the zero subscripts represent the initial conditions. The indicated air speed, being proportional to the square root of the density, will remain constant with increased altitude for constant L/D .

Most of the computations made for this study were based on flight at constant L/D because of the general familiarity with that condition and also because of the labor involved. Contrary to common understanding, however, flight at constant L/D does not necessarily result in the greatest range for a given average speed; a later analysis will show that flight at either constant speed or constant power may be better. Since the results are given mostly for comparative purposes, the assumed method of flight does not change the relative values.

For long-range airplanes the fuel weight constitutes a large proportion of the total weight. As the fuel is used up, the speed of flight must be reduced accordingly if the L/D is to remain constant. If it be desired that the true air speed remain constant also, it becomes necessary to increase the height as the weight decreases thus:

$$\frac{W}{W_0} = \frac{\rho}{\rho_0}$$

In reference 3 Oswald derives some convenient equations for calculating L/D based upon the fundamental airplane parameters,

$$\frac{D}{L} = 0.002558 \frac{\sigma}{l_p} v^2 + 124.4 \frac{l_s}{\sigma v^2}$$

where $\sigma = \frac{\rho}{\rho_0}$, the relative density

$l_p = \frac{W}{f}$, parasite loading (lb./sq.ft.)

$l_s = \frac{W_s}{b_e^2}$, effective span loading (lb./sq.ft.)

v , air speed (miles/hour)

f , equivalent parasite area (sq.ft.)

$b_e = e^{1/2}(kb_1)$, effective span (ft.)

e, airplane efficiency factor

k, Munk's span factor

b_1 , largest individual span of the wing
cellule (ft.)

If the performance characteristics of the airplane under consideration are known, the value of f can be easily obtained from the following equation given in reference 4:

$$f = \frac{(b \cdot hp_0 \times \eta_{\max} - \frac{2W^2}{\pi \rho b e^2 V_{\max} 807}) 349.5}{\rho V_{\max}^3}$$

From reference 3,

$$\left(\frac{L}{D}\right)_{\max} = 0.886 \left(\frac{l_p}{l_s}\right)^{1/2}$$

$$V \text{ for } \left(\frac{L}{D}\right)_{\max} = 14.85 \left(\frac{l_s - l_p}{0.175}\right)^{1/4} = 14.85 \times \frac{1}{0.175^{1/4}} (2, 2p)^{1/4}$$

A convenient chart plotted from these two equations is given in figure 1 for obtaining $\left(\frac{L}{D}\right)_{\max}$ and the air speed for $\left(\frac{L}{D}\right)_{\max}$.

Figure 2 is a portion of the same chart with points plotted from data for several relatively large airplanes and flying boats, representing the best examples of present-day design. It may be noted that the $\left(\frac{L}{D}\right)_{\max}$ is fairly high for all of them. Considering the high speeds of these airplanes, which range from 150 to 240 miles per hour, the speed for $\left(\frac{L}{D}\right)_{\max}$ is unfortunately very low. Of course, the speed for $\left(\frac{L}{D}\right)_{\max}$ may be increased by flying at high altitudes, or by increasing the design weights.

Figure 3 shows the effect of changing the loading of a modern transport airplane. The loading may be considered as being changed in either of two ways: The airplane

may be designed for different gross weights with the geometrical dimensions held constant, or the weight may be held constant and the dimensions scaled up or down. For the sample computations carried out later in this analysis the airplane weight is held constant in order that the specific engine weight may also remain constant.

The effect of decreasing the relative air density by increasing the altitude has essentially the same effect as changing the loading, as may be seen from figure 4. It may be noted that flight at an altitude of about 22,000 feet has the same effect on the air speed for a given value of L/D as doubling the normal loading.

Assuming other factors constant a decrease in span loading will increase the $\left(\frac{L}{D}\right)_{\max}$, and the speed for $\left(\frac{L}{D}\right)_{\max}$ will be slightly decreased. (See fig. 5.) If the airplane is to be flown at speeds substantially above the speed for $\left(\frac{L}{D}\right)_{\max}$ there is little advantage in increasing the span except for take-off and climb. If the parasite area is also increased by so doing, as is almost inevitable, the resulting L/D at these higher speeds may be actually reduced. (See fig. 6.)

On the other hand, if the airplane is to be flown at approximately the speed for $\left(\frac{L}{D}\right)_{\max}$, increasing the span will be beneficial whether it be done by increasing the aspect ratio without affecting the wing area or by increasing the span and chord in proportion. Aside from L/D considerations, increasing wing area results in better take-off characteristics and decreased landing speeds, which are of importance for long-range airplanes.

Propeller considerations.— It can be shown that if the airplane is flown at a constant angle of attack the propeller will likewise operate at a constant value of V/nD ; consequently, the propulsive efficiency will remain constant. This condition is fortunate for it is then not necessary for flight at constant L/D to change the pitch of the propeller during flight except for take-off.

There are three important considerations in selecting propellers for long-range airplanes. First, the propul-

sive efficiency at cruising speed should be as high as possible; second, the propeller should absorb the power at a high value of engine torque in order to take advantage of low specific fuel consumption; third, the propeller should be of the controllable type and be as large in diameter as possible consistent with efficiency in order that the take-off thrust will be as high as possible.

The first two requirements can be fulfilled fairly well with a fixed-pitch propeller. It should be noted, however, that as fuel is used up and the weight decreases the engine torque decreases proportionately if the flight is at constant L/D and V/nD . Decreasing the torque will have the effect of increasing the specific fuel consumption. It might be thought that a controllable propeller could be used to advantage in adjusting the pitch during flight to maintain constant torque. An investigation of the operating characteristics for flight at constant L/D and also at constant torque reveals that the revolution speed, neglecting changes in propulsive efficiency, would decrease rapidly with decreasing weight as:

$$\frac{N}{N_0} = \left(\frac{W}{W_0} \right)^{3/2}$$

Furthermore, since
$$\frac{V}{V_0} = \left(\frac{W}{W_0} \right)^{1/2}$$

then
$$\frac{\left(\frac{V}{nD} \right)}{\left(\frac{V}{nD} \right)_0} = \frac{W_0}{W}$$

where N is the engine revolution speed. The formula indicates that for long-range airplanes the pitch would have to be increased to extremely high values in order that the propeller absorb the power at the low engine speeds. The actual pitch may be determined from torque or power coefficient curves thus:

$$\frac{C_P}{C_{P_0}} = \frac{C_Q}{C_{Q_0}} = \left(\frac{N_0}{N} \right)^2$$

because

$$C_Q = \frac{C_P}{2\pi}$$

and

$$\frac{C_P}{C_{P_0}} = \left(\frac{W_0}{W} \right)^3$$

where C_P is the power coefficient of the propeller and C_Q is the torque coefficient.

It can be seen that with the power coefficient increasing at this rate the pitch would soon be increased beyond the point for maximum efficiency and even into the range wherein the blades would be stalled.

The ideal pitch is, of course, the one in which the ratio of propulsive efficiency to specific fuel consumption would be a maximum. It can be shown by a similar analysis, however, that this optimum pitch remains substantially constant for all values of weight provided that the airplane be flown at a constant L/D . This condition probably varies somewhat for different airplanes and flying conditions, but there is no evidence to indicate that any material gain can be had by changing the pitch during flight because the propulsive efficiency drops with increased torque, in the usual case, about as fast as does the specific fuel consumption.

Airplanes that are normally flown at high speeds will be materially handicapped for long flights at reduced speeds unless the propeller pitch and diameter are changed to provide a reasonably high engine torque, in order that the specific fuel consumption remain fairly low.

For the computations carried out in this paper the propellers were selected in accordance with the data furnished in references 5 and 6 for fuselage 6.

Power Plant Considerations

For flight at constant L/D and V/nD the following relations hold true:

$$\left(\frac{P}{P_0} \right) = \left(\frac{W}{W_0} \right)^{3/2} \left(\frac{\rho_0}{\rho} \right)^{1/2}$$

$$\frac{Q}{Q_0} = \frac{W}{W_0}$$

$$\frac{Q}{Q_0} = \text{constant, with changes in density}$$

$$\left(\frac{N}{N_0}\right) = \left(\frac{W \rho_0}{W \rho}\right)^{1/2}$$

where P is the power and Q , the engine torque.

For flight at high altitudes the engine power must necessarily be greater than at sea level because the speed of flight at constant L/D will be higher. The greater engine weight accompanying the greater power necessary for altitude flying will, assuming constant gross weight, result in less fuel weight being carried; consequently, the range will be slightly less. On the other hand, the speed of flight at altitudes will be much greater and the larger engines required for altitude flight will enable the airplane to take off more readily. The increased engine weight necessitated by altitude flight will be more pronounced for unsupercharged engines than for supercharged engines, as will be brought out later. The effect on the take-off will also be more pronounced for the unsupercharged engine.

Specific fuel consumption.— The relationship between torque and engine speed is quite important from considerations of specific fuel consumption. In figure 7 average specific fuel-consumption curves for a number of unsupercharged engines have been plotted. Engine tests ordinarily include only a full-throttle curve and a propeller-load curve in which the torque is proportional to N^2 . Since it was desirable in this analysis to vary the engine torque and speed at will to cover a number of variables, it became necessary to formulate a reasonable chart covering the engine speed and torque range. The fuel-consumption curves for different values of torque were assumed to be parallel to the full-throttle curve, which ordinarily is nearly constant over a wide range. Engine tests are being made to substantiate this hypothesis. Even if these curves are not closely characteristic of the average engine curves, the results of this study will not be seriously affected for they are only relative.

For all long-range flights the fuel consumption has been the subject of deep concern because, except where fuel-flow or air-fuel meters have been installed, it has

been difficult to make accurate checks on the fuel consumption while in flight. Until the recent introduction of automatic mixture controls bench test conditions have been different from those in flight with the result that the flight fuel consumption usually has been higher than anticipated. The effect of the fuel-air ratio on power and economy is given in figure 8.

If the fuel-air ratio is maintained constant with increased altitude and the torque is also held constant by opening the throttle, or by means of a supercharger, the fuel consumed per mile will also remain substantially constant because the work done is independent of density. If the engine is supercharged, the power required by the supercharger must, of course, be added to the power required for flight, which will result in a higher fuel consumption.

Pilots are usually reluctant to lean the mixture any great amount for fear of overheating the engine owing to the slow combustion accompanying lean mixtures. Until the recent introduction of a fuel-air indicator and automatic mixture control, there was no satisfactory method of determining the fuel-air ratio in flight. The usual procedure was to lean the mixture until the engine speed started to fall off or until it became rough owing to uneven firing. Tests in Great Britain (reference 8) have shown that engines can be run on much leaner mixtures than is commonly supposed without signs of damage to the engine. An air-cooled engine was run at a fairly high value of torque and engine speed for 100 hours at a specific fuel consumption of 0.48 without damage, and a water-cooled engine was operated on a fuel consumption of 0.43 under similar conditions. It was found possible to run on much leaner mixtures for throttled conditions than for full throttle.

Improvements, particularly in the cooling of late types of American air-cooled engines, have enabled the fuel consumption to be reduced to about 0.42 and less.

In the British tests (reference 8) there was evident a noticeable improvement in fuel economy by advancing the ignition timing for lean mixtures (fig. 9).

The effect of compression ratio on fuel consumption is well known. Diehl (reference 1) gives the relation $c = 0.75 - 0.4 \text{ C.R.}$ for service engines of several years ago.

Tests on radial engines equipped with fuel-injection systems (reference 9) have indicated a minimum specific fuel consumption of the order of 0.42. The maximum power was increased about 15 percent. An increase in power should result in a lower specific weight, provided that the injection system does not weigh appreciably more than the carburetor system.

Altitude operation.— On account of the material gain in speed of flight at altitudes without an appreciable loss in range, the behavior of engines at altitudes warrants some discussion. The variation in engine power in a standard atmosphere for unsupercharged engines is often given by the expression:

$$\frac{P}{P_0} = \left(\frac{\rho}{\rho_0} \right)^n$$

The exponent n has been given values ranging from 1.12 to 1.3 by different authorities. For the purpose of this paper the exponent 1.3 has been chosen. (See figure 10.)

Engines are sometimes rated at powers higher than those at which they may be safely operated; they must therefore be throttled for sea-level operation for unsupercharged engines or for operation at the critical altitude for supercharged engines. As far as mean effective pressure or torque is concerned, the same effect may be obtained by flying at a somewhat higher altitude than that at which the engine is rated. The full-throttle torque of an unsupercharged engine would be lowered to 82.5 percent of the value at sea level for flight at 5,000 feet. The propeller pitch must, of course, be adjusted at that altitude to a value such that the full-throttle engine speed does not exceed the allowable cruising value. Ordinarily, cruising power of 75 percent rated power is considered reasonable with the engine speed held at 91 percent of the rated value. Throttling an unsupercharged engine at sea level to 91 percent rated speed with a fixed-pitch propeller will give approximately these conditions. In view of this decrease in engine power with altitude it may be desirable to fly at such a height that the full-throttle power will equal the desired cruising power, on account of the resulting higher speeds without

any loss in range, assuming a constant fuel-air ratio.

For long-range airplanes the altitude may be increased as the weight is decreased because less power is required. The effect of increasing the altitude with decreasing weight will result in flight at more nearly constant speed for constant L/D , instead of a decreasing speed for flight at the same altitude. The explanation is shown by the following relations:

$$\left(\frac{P}{P_0}\right)_{\text{req}} = \left(\frac{W}{W_0}\right)^{1.5} \left(\frac{\rho_0}{\rho}\right)^{0.5} \quad \text{for constant } L/D$$

and

$$\left(\frac{P}{P_0}\right)_{\text{avail}} = \left(\frac{\rho}{\rho_0}\right)^{1.3} \quad \text{for unsupercharged engines}$$

If the altitude is increased as the weight diminishes in such a way that the power required equals the cruising power available, then

$$\left(\frac{W}{W_0}\right)^{1.5} \left(\frac{\rho_0}{\rho}\right)^{0.5} = \left(\frac{\rho}{\rho_0}\right)^{1.3}$$

or

$$\left(\frac{\rho}{\rho_0}\right) = \left(\frac{W}{W_0}\right)^{0.834}$$

and

$$\frac{V}{V_0} = \left(\frac{W}{W_0}\right)^{0.083}$$

which shows that the true air speed will be nearly constant even though the weight decreases, as contrasted with the relation,

$$\frac{V}{V_0} = \left(\frac{W}{W_0} \right)^{0.5}$$

for flight at constant L/D and altitude.

Assume, for example, a long-range airplane carrying 50 percent of its weight in the form of fuel. If the altitude were increased as the weight decreased according to the foregoing assumptions the airplane would finish the flight at an altitude at which the relative density was 0.56, corresponding to about 18,700 feet. The velocity would be 0.94 of the velocity at the start of the flight. Had the flight been at sea level, the velocity would be only 0.71 of the initial velocity.

The same system could be applied for supercharged engines; the only difference would be that the critical altitude would be equivalent to sea level for the unsupercharged engines.

Perhaps the best single method for increasing cruising speeds at the present time without materially decreasing the range is by means of supercharging the engines for fairly high-altitude flight. This method, however, is not without certain disadvantages. The supercharger absorbs a certain amount of the power developed, which affects both the fuel consumption and the engine weight because the engines must be larger to supply power for the supercharger. As the fuel consumption is in proportion to the total power developed, the net power applied at the propeller must be corrected for the power absorbed by the supercharger. Data taken from reference 10 have been plotted (fig. 11) in the form of a supercharger efficiency factor λ , which is the ratio of the power applied to the propeller to the total power developed. The specific fuel consumption can be corrected for this supercharger power by dividing it by the efficiency factor.

Because the supercharger absorbs a part of the engine power, the engine must be increased in size and consequently in weight in order that sufficient power may be developed for both the supercharger and propeller. For the geared centrifugal supercharger the engine weight should be increased about 8 percent for a critical altitude of 30,000 feet and less for lower altitudes (reference 10).

For the turbocentrifugal supercharger the engine size need not be increased at all because the increased power developed due to the decreased back pressure approximately equals the power required to drive the supercharger.

Another important supercharger characteristic which affects the performance of long-range airplanes depends on the control at altitudes below the critical altitude. For the turbocentrifugal supercharger the control is almost ideal; the exhaust gases are passed through the turbine only to the extent that is needed to maintain the desired power and the take-off power is not decreased by the supercharger.

For the geared types of supercharger, full-throttle operation at sea level may increase the manifold pressure to prohibitive values, necessitating part throttle operation and lower power outputs for take-off and climb.* The present tendency is, however, to allow the manifold pressure to go beyond the normal rating for short periods, which results in a much higher power available for take-off.

With controllable propellers both the engine speed and manifold pressure can be regulated to give their respective desired values. The only loss, then, at low altitudes is that due to the power required to drive the supercharger. For the geared centrifugal type the power required to drive the supercharger is nearly constant up to the critical altitude (reference 10) and amounts to about 20 percent of the gross sea-level power for the supercharged engine with a critical altitude of 30,000 feet.

Engine weights.— The relationship between engine specific weight and horsepower is plotted in figure 12 for a number of present-day engines. Even though the plot for the spark-ignition engines includes all types there seems to be little dispersion. Of course, the basic design characteristics are about the same for all of the engines, except for the geometric lay-out of the cylinder locations. In the case of the liquid-cooled engines, additional weight must be added for the cooling systems. Air-

*This problem is being solved, however, by several means: two-stage superchargers having a clutch for one stage; two-speed gear trains with means for shifting gears; and combinations of exhaust-driven and gear-type centrifugal superchargers, the exhaust-driven type being used only at high altitudes.

planes of sufficient size to utilize large engines of low specific weight are better adapted for long flights than airplanes using small engines because of the better disposition of the weights.

Compression-ignition engines.— There is a field for compression-ignition engines for long-range airplanes because of their inherently high thermal efficiency. The usual relative minimum specific fuel consumption of spark- and compression-ignition engines is of the order of 0.49* and 0.37, respectively, a reduction of 25 percent (fig. 13). Furthermore, the fuel consumption does not increase as rapidly with decreasing power for the compression-ignition engine as it does for the spark-ignition engine. The power is reduced, in the case of the compression-ignition engine, by reducing the fuel-air ratio.

Compression-ignition engines are inherently heavier than spark-ignition engines. (See fig. 12.) The maximum cylinder pressures are higher for the compression-ignition engine, necessitating heavier parts to withstand the loads; the mean effective pressures are much lower, requiring a larger displacement for the same power; and the running speeds are, in general, somewhat lower.

One possibility in connection with minimizing the differences in weights of the two types of engines should not be overlooked. Compression-ignition engines may be operated two-stroke without materially sacrificing fuel economy but spark-ignition engines ordinarily cannot be. Air from a blower can be used for scavenging the two-stroke compression-ignition engine; whereas, a mixture of air and fuel is ordinarily required for the two-stroke spark-ignition engine. There may be some loss in mean effective pressure and in running speed when operating two-stroke; but the increased number of firing strokes would much more than make up for this loss. It appears possible, therefore, that with more development, two-stroke compression-ignition engines might be built for nearly the same weight as four-stroke spark-ignition engines.

Even though the weights of compression-ignition engines are greater than those of spark-ignition engines the lower fuel consumption may more than balance the added weight for relatively long flights. The range at which compression-ignition and spark-ignition engines are equal

*Recent improvements in spark-ignition engines together with the use of higher octane fuels have reduced this value considerably.

in merit depends upon their relative weights and fuel consumptions. Such a balance is made for the general case in figure 14, which was taken from reference 11. These curves are mathematically derived with the Breguet range formula as a basis. The symbols used are consistent with those used elsewhere in this paper with modifications to differentiate between the two types of engine installations, as

c_1 , specific fuel consumption for the spark-ignition engine.

c_2 , specific fuel consumption for the compression-ignition engine.

d_1 , specific engine weight for the spark-ignition engine.

d_2 , specific engine weight for the compression-ignition engine.

t , specific tank weight, lb./lb. fuel.

P_a , total brake horsepower available.

W_{f1} , weight of the fuel for the spark-ignition engine.

Of the three parameters in the chart c_2/c_1 is a function of relative fuel consumption, $\frac{d_2 - d_1}{\frac{W}{P_a} (1 + t)}$ is a function

of the relative engine weight, and W_{f1}/W is a function of the range. It may be readily seen that for any ratio of fuel consumption there will be a definite range at which the effect of the differences in the weights of the two engines will be zero.

Structural Weight

In any airplane design the economics involved in balancing weight against drag is of the first order of importance. Since range is, in a sense, a criterion of overall efficiency the problem becomes acute for long-range airplanes. Certain types of design lend themselves to low structural weight but relatively high drag, and vice versa. Furthermore, although small airplanes may be built effi-

ciently when designed in a certain manner, a large one built similarly might be relatively inefficient. The problem therefore becomes one involving both the type of structure and the size.

In figure 15 a balance is made between the W_e/W_o and L/D for two airplanes of equal range

where W_o is the gross weight

W_e , the gross weight less fuel

The subscripts a and b denote the two different airplanes of equal range.

Example: When comparing a cantilever monoplane and a braced monoplane the following conditions might be found to exist. From wind-tunnel tests the $(\frac{L}{D})_a$ of the braced monoplane is found to be 12 and that for the cantilever $(\frac{L}{D})_b$ is 15. If $(\frac{W_e}{W_o})_a$ for the braced monoplane can be designed for a value of 0.5, then the value for the cantilever need be 0.575 for equal range. If the cantilever value turns out to be greater than this amount, the advantage lies with the braced monoplane. If it be found that the cantilever monoplane weight is 0.65, the ratios of the values of $(\frac{L}{D})$ would be 1.61 for the same range. Or, the $(\frac{L}{D})_b$ of the cantilever would have to be 1.61×12 , or 19.3, in order that the two airplanes be of equal range. But, since the cantilever has an $(\frac{L}{D})_b$ of only 15, the range would be $\frac{15}{19.3}$, or 0.78 of the range of the braced monoplane.

The chart of figure 15 affords a simple method of comparing airplanes of different types with respect to range, if the weights and lift-drag ratios are known. It also illustrates the relative importance of weight and drag. From the example given above an increase in W_e/W_o from 0.5 to 0.575 amounts to 15 percent increase in weight (less fuel); this increase must be balanced by an increase in L/D from 12 to 15, which amounts to 20 percent reduction in drag. From this result it appears that the weight (less

fuel) is a more important consideration than drag. The magnitude of relative importance of these quantities will depend upon the particular airplane under consideration.

An analysis of the range possibilities of several airplanes is made in the table (p. 2Q). Although the values may be found useful and interesting, the airplanes listed are not closely comparable because of the different weights and purposes for which they were designed. In addition, the values are subject to a certain amount of error because of the uncertainty of the performance figures and because of the assumption necessary for computing the effective span.

The gross weights of the airplanes listed range from 5,250 to 51,000 pounds. It is interesting to note that the ratio of the weight empty to gross weight is not greatly different for these two extremes. The $\left(\frac{L}{D}\right)_{\max}$ is surprisingly high for most of the airplanes. Several of the values were checked with wind-tunnel tests and the agreement was found to be good. The speed for $\left(\frac{L}{D}\right)_{\max}$ averages about 100 miles per hour for all the airplanes, which is relatively low considering their cruising speeds. The potential range factor R_P is a measure of the relative ranges of the various airplanes based on the data given and is defined as:

$$R_P = \left(\frac{L}{D}\right)_{\max} \times \eta_{\max} \times \log_{10} \frac{W_0}{W_e}$$

One of the large commercial flying boats (airplane 18) has a longer potential range than one of the specially designed long-range airplanes (airplane 22). The advantage of the flying boat lies mostly in its very light structural weight, for the $\left(\frac{L}{D}\right)_{\max}$ is somewhat lower.

The indications are that, since the largest airplane (or flying boat) listed is also about the most economical structurally, the large sizes may lend themselves to more efficient design. There is probably a limit to which the size may be increased without increasing the weight proportionately, but that limit is not evident at present.

POTENTIAL RANGE CHARACTERISTICS FOR SEVERAL AIRPLANES

Airplane	Gross weight lb.	Wt. empty Wt. loaded	l_s lb./sq.ft.	l_p lb./sq.ft.	$(L/D)_{max}$	V for $(L/D)_{max}$ m.p.h.	Possible max η	R_p potential range factor
Low-wing cantilever monoplane, single-engine transport								
1	8,500	0.63	4.10	1,145	14.8	122	0.87	2.60
2	5,800	.57	3.53	883	14.0	111	.87	2.94
3	7,000	.58	3.37	731	13.0	105	.86	2.61
4	7,000	.50	3.37	682	12.6	103	.86	3.26
5	7,100	.47	3.42	761	13.3	106	.86	3.78
6	8,500	.62	3.36	677	12.6	103	.85	2.18
Low-wing cantilever monoplane, two-engine transport								
7	13,650	0.66	2.77	641	13.5	97	0.87	2.18
8	9,400	.61	3.45	884	14.2	110	.87	2.66
9	17,500	.66	2.69	784	15.1	101	.86	2.32
Low-wing cantilever monoplane, two-engine bomber								
10	12,830	0.60	2.87	592	12.7	95	0.86	2.42
Low-wing braced monoplane, three-engine transport								
11	8,750	0.65	2.70	450	11.4	88	0.86	1.83
High-wing cantilever monoplane, two-engine transport								
12	19,840	0.61	2.72	572	12.8	93	0.84	2.3
High-wing braced monoplane, single-engine transport								
13	5,600	0.69	2.42	467	12.2	67	0.86	1.68
14	11,400	.47	2.86	583	12.6	95	.82	3.45
15	13,000	.56	2.05	427	12.8	81	.83	2.63
High-wing braced monoplane, two-engine bomber								
16	16,330	0.50	3.02	558	12.0	95	0.85	3.06
High-wing braced monoplane, four-engine flying boat								
17	38,000	0.52	3.24	693	12.9	102	0.85	3.12
18	51,000	.45	3.35	734	13.1	105	.85	3.81
Biplane, four-engine flying boat								
19a	34,800	0.59	2.75	348	10.0	83	0.80	1.83
19b	42,500	.48	3.35	425	10.0	91	.80	2.54
19c	48,000	.43	3.78	480	10.0	87	.80	2.97
Biplane, two-engine transport								
20	16,800	0.67	2.32	517	13.2	88	0.83	1.89
Special long-range racing monoplane, two engines								
21	5,250	0.54	3.39	1,255	17.0	120	0.84	3.77
Special long-range cantilever monoplane, one engine								
22	17,000	0.55	2.81	855	15.5	104	0.86	3.45
23	17,196	.49	2.42	1,050	18.5	105	.85	4.87

II - THE EFFECT OF DIFFERENT VARIABLES ON THE RANGE OF A SAMPLE AIRPLANE

In the preceding sections an attempt was made to analyze the various factors affecting range and to evaluate them relatively. As the problem not only includes the individual factors but all of them taken collectively, it becomes necessary to determine their mutual relationships when embodied in an airplane. It is not always possible to evaluate the relative importance of the different variables for the general case when taken collectively because of their interacting relationships. In order to illustrate the effects, it then becomes necessary to assume a special case and systematically to change the variables. The results are, in general, qualitative, being strictly quantitative only for airplanes similar to the one assumed.

Assumptions and Methods

In order that the assumptions be reasonable, the airplane assumed was patterned after an existing type with possibilities for long range. With this airplane as a basis the variables have been changed as desired in order to illustrate the effect on the airplane as a whole. The normal airplane assumed has the following basic characteristics:

Gross weight, $W_0 = 17,500$ lb.

Span loading, $l_s = 2.69$.

Parasite loading, $l_p = 784$.

The power loading was determined by the requirements of flight for each example.

The airplane has two engines, the weights of which were determined from figure 12.

The specific fuel consumption was determined from figure 7 or 13.

The propellers are two-blade, direct-drive, and were selected to give the highest efficiency at the designed cruising speed, except for the cases noted.

The L/D curves of the airplane are given in figures 3, 4, 5, and 6 for various flight conditions and loadings.

The method employed for computing the range is based on the graphic integration of the basic range formula. The following table shows sample computations for determining miles per pound of fuel for different periods of the flight or for different fuel weights carried.

SAMPLE COMPUTATIONS SHOWING THE METHOD EMPLOYED FOR
DETERMINING RANGE FOR DIFFERENT FUEL LOADS

(Initial speed of flight, 100 m.p.h.; flight at constant L/D of 15.1; initial engine power, 0.75 rated power; propulsive efficiency, 0.78)

$\frac{W}{W_0}$	Fuel consumed	$\frac{Q}{Q_{rated}}$	$\frac{N}{N_{rated}}$	Specific fuel consumption, c	D drag	Miles $\frac{1 \text{ lb. fuel}}{(375 \eta / cD)}$	Range (from integration)
	Pounds			$\frac{\text{Pounds}}{\text{b.hp.-hr.}}$	Pounds		Miles
1.0	0	0.825	0.910	0.486	1,160	0.518	0
.9	1,750	.743	.862	.497	1,044	.564	950
.8	3,500	.660	.814	.512	929	.614	1,985
.7	5,250	.578	.760	.545	813	.660	3,090
.6	7,000	.495	.705	.581	696	.723	4,280
.5	8,750	.413	.643	.628	580	.800	5,600
.4	10,500	.330	.576	.690	463	.911	7,070

In figure 16 sample differential curves are shown for different flying speeds. In figure 17 the fuel load required for different flight distances was plotted for different values of flying speed. It may be noted that since the computations were generally made for flights at constant L/D the speed diminished as the fuel decreased and only the speed for the beginning of the flight is given.

The weights of the engines, the oil, and the tanks are important variables in an analysis of this type and must be taken into account when determining the amount of fuel that can be carried. The weight allotted for engines, fuel, oil, and tanks was fixed for any airplane. The engine weight was determined by the maximum power required; the weights of the oil and tanks were determined by the quantity of fuel carried, which was in turn determined by the amount of remaining weight available. The procedure followed in determining the fuel available, and consequently the range, was one in which various weights were systematically added or subtracted. The following table illustrates this procedure.

SAMPLE COMPUTATION ILLUSTRATING THE METHOD EMPLOYED
FOR DETERMINING FUEL AVAILABLE FOR DIFFERENT
VALUES OF ξ

(Initial speed of flight, 100 m.p.h.)

ξ	A-(B+C+D+E)	C	A-(B+D+E)	B+D+E	Fuel = 0.9(B+D+E)
	Pounds	Pounds	Pounds	Pounds	Pounds
0.9	15,750	1,180	16,930	570	513
.8	14,000	1,180	15,180	2,220	2,000
.7	12,250	1,180	13,430	4,070	3,660
.6	10,500	1,180	11,680	5,820	5,240
.5	8,750	1,180	9,930	7,570	6,820
.4	7,000	1,180	8,180	9,320	8,390

where ξ is defined as

$$\xi = \frac{A - (B + C + D + E)}{A}$$

in which

A is gross weight
B, weight of fuel
C, weight of the engines
D, weight of the tanks
E, weight of the oil

The term ξ is a structural efficiency factor because the numerator is nearly all structural weight, or structural weight and cargo. The smaller the structural weight, the greater the cargo for a given value of ξ . For simplified purposes of illustration, it might be assumed that the numerator is always nothing more than structural weight.

For given values of ξ the structural and cargo weights were fixed, leaving a definite residual weight for the engines, tanks, oil, and fuel. The engine weights were then added to the structural and cargo weights. After subtracting these values from the gross weight there remained available weight for fuel, oil, and tanks. The fuel weight available was assumed to be equal to 0.90 of the weight available for the fuel, oil, and tanks. The value 0.90 is fairly representative for the average condition.

The range was determined for different values of (fig. 18) by taking different values of fuel available from the fuel-required curves. The method employed for determining the time required for flight was about the same as that for range. Figure 19 shows sample graphic integration curves; figure 20 shows sample time-required curves.

Example 1, High-Speed Airplane Adapted for Long Range

Design and flight conditions:

Sufficient power to fly at about 200 miles per hour.

Normal span and parasite loadings.

Flight at constant L/D at sea level.

It was assumed that the airplane was intended for high-speed flight and that the power available was greatly in excess of the requirements for flying at the speed for maximum range with the result that the large engine weights reduced the weight available for fuel. In figure 21 the range at different speeds of flight and different values of ξ is plotted for different fixed-pitch propeller designs. It may be noted that there is a marked improvement in range at low speeds when the high-speed propellers are replaced by those which absorb the power at higher torque values because the specific fuel consumption is reduced.

The propulsive efficiency is little affected. With the high-torque propeller it will not be possible, of course, to fly at 200 miles per hour. Controllable propellers designed for high speeds could not be used for the purpose of increasing the torque at greatly reduced speeds unless means were incorporated to change the gear ratio also.

Example 2, Comparison between Flight at Constant L/D,
Speed, and Power

Design and flight conditions:

Sufficient power to cruise at 0.75 rated power at the initial part of the flight.

Normal span and parasite loadings.

Flight at sea level.

In the first part of this paper it was pointed out that flight at constant L/D does not necessarily result in the greatest range for a given average speed but that flight at either constant speed or power may be better. In figure 22 a comparison of range is made between these three flight methods on the basis of the designed initial speed of flight. It may be noted that the curves peak at about the same value of range but that the peaks occur at different designed initial flight speeds, especially for the longer range conditions. If it be remembered that the speed diminishes as the fuel is used up for flight at constant L/D, that the speed is constant for flight at constant speed, and that the speed increases for flight at constant power, it can be seen that the curves are displaced from what they would be had the average speed been used as a basis for comparison. For example, if a comparison is made on the basis of average speed, the curves for flight at constant speed would remain fixed; whereas the curves for flight at constant L/D would be displaced to correspond to the lower values of average speed and the curves for flight at constant power would be displaced to correspond to the higher average speeds.

It was found convenient in making this correction for the differences in average speed to plot range against elapsed time of flight (fig. 23). These curves indicate

that, for any given elapsed time, a greater maximum range can be obtained by flying at constant power than for either constant speed or constant L/D , except for the extreme condition where a slight advantage lies with flight at constant speed. It should be noted that in order to accomplish this gain in range for a given elapsed time the initial speed must be lower for the conditions of flight at either constant speed or power than for constant L/D .

The advantage of flying at either constant speed or power is further illustrated in figure 24, which is a comparison based on the average rather than on the initial speed. It probably would be found, in practice, that flight at constant speed is preferable to either of the other two methods because of the greater convenience. Flight at constant L/D necessitates decreasing the speed, while flight at constant power necessitates increasing the speed and at the same time increasing the propeller pitch to maintain constant engine operating conditions. Although controllable propellers would be necessary for flight at constant power, this feature did not account for all the gain for it can be seen that the condition for flight at constant speed, which might be made with fixed-pitch propellers, is nearly as good. The advantage of the conditions of flight at constant speed or constant power over that for constant L/D may be explained on the basis of specific fuel consumption and L/D . From figure 25 it can be seen that, for an average speed of flight of 140 miles per hour, the specific fuel consumption increases as the weight diminishes for flight at constant L/D but remains nearly constant for the other two conditions. On the other hand, the L/D decreases as the weight decreases for flight at either constant speed or power but, since the average speed is the same for all conditions, the average value of L/D is nearly the same. The propulsive efficiency remains about the same for all conditions and it is therefore evident that the lower average specific fuel consumption accounts for nearly all the gain due to flying at either constant speed or power. If engines were used in which the specific fuel consumption did not increase as rapidly with decreasing power as was assumed for these examples, the differences in range would not be as great as indicated here, and the order of merit might even be changed.

Example 3, Flight at Altitudes

Design and flight conditions:

Sufficient power to cruise at 0.75 rated power at the initial part of flight.

Normal span and parasite loadings.

Flight at constant L/D .

Unsupercharged engines.— In order to fly at altitudes with unsupercharged engines, the engine size must be increased to offset the loss in power due to the decreased density. If it is intended that the engines cruise at a fraction of their rated sea-level power, any desired amount of effective throttling may be accomplished by cruising at an altitude such that full-throttle power equals the desired cruising value. For example, flight at 5,000 feet will reduce the full-throttle torque to about 0.825 of its sea-level value (fig. 10) and, if the engine speed is held to 0.91 of its rated value by means of adjusting the propeller pitch, the power will be 0.75 of its rated value. If it were desirable to fly at constant L/D and also constant power output, it would be necessary to increase the altitude as the weight decreases in order that the required power equal the full-throttle power available. This procedure would also result in flight at nearly constant speed. It has been shown, however, that flight at either constant speed or power might result in greater range for most speeds without the added inconvenience of changing the altitude.

Figure 26 illustrates the effect of designing for flight at altitudes with unsupercharged engines. It may be noted that the gain in speed for a given range is limited to an altitude of 5,000 feet because the engine size must be increased for higher altitudes. At altitudes above 5,000 feet the gain due to the lower drag for a given speed is more than offset by the added weight of the engines and the lower specific fuel consumption except for high-speed conditions where the factors nearly balance.

If it were necessary to increase the engine size in order to improve the take-off, little if any range would be sacrificed thereby when the engines were effectively throttled the desired amount by flying at the appropriate altitude. (See fig. 26.)

Supercharged engines.- In figure 27 the range is given for various speeds and altitudes for the airplane equipped with supercharged engines. The superchargers were assumed to be of the geared centrifugal type capable of compressing the air to sea-level pressure at the altitude of flight. The fuel consumption was corrected for the power absorbed by the supercharger by the factor given in figure 11. Engine-weight correction factors (taken from fig. 4 of reference 10) were applied to account for the increased engine size necessary to operate the supercharger. No account was taken of the supercharger weight because of its intangible nature. This type of supercharger is ordinarily built into the engine and acts as a rotary distributor for radial engines. Unless air intercoolers are employed the engine weight is ordinarily not greatly increased. For other types of engines, the supercharger would probably add weight equal to the weight of the isolated supercharger.

It may be noted from figure 27 that the maximum possible range at low speed is somewhat reduced with increasing altitudes but that the range at higher speeds is increased by increasing the altitude of flight. For any desired cruising speed there appears to be an optimum altitude at which the range is a maximum and this altitude increases with speed. The maximum optimum altitude, or the greatest altitude at which any increased range could be realized, is not reached on the chart even though the altitudes extend to 40,000 feet. The gain is decreased, however, as the altitudes increase to the extreme values given and, if the mechanical difficulties which would be incurred by reaching those altitudes were considered, the highest practicable altitude would probably be somewhat less than 40,000 feet. In this analysis no account is taken of the weight of high-altitude equipment that would be necessary for altitudes above 15,000 or 20,000 feet, nor of the fuel used to climb. The only energy lost in the climb would be that required to lift the fuel to the cruising height, for the energy expended to raise the airplane proper and the cargo would be substantially regained upon descending to the ground.

It should be pointed out that, if a more efficient supercharger had been assumed, less range would have been sacrificed at all altitudes above sea level. A turbocentrifugal supercharger is more efficient for high altitudes but its weight and air resistance is ordinarily a definite handicap. Geared centrifugal superchargers have been gen-

orally limited to low altitudes because of mechanical complications.

Figure 28 illustrates the saving in power accomplished by flying at various optimum altitudes corresponding to different speeds of flight. This saving in power due to the lower drag at reduced air density accounts for most of the increase in range at relatively high speeds. Both the fuel consumption and engine weights are less for the altitude conditions than for the sea-level condition, even though additional power is required to drive the supercharger. It seems evident that if operating costs were considered there would be a material saving by flying at high altitudes. Considering the cost, the maximum optimum altitude may be different from that indicated for range.

Example 4, Variations Made in Span and Parasite Loadings

Design and flight conditions:

Sufficient power to cruise at 0.75 rated power at the initial part of flight.

Flight at constant L/D at sea level.

A decrease in equivalent span loading by means of increasing the span without affecting the parasite area materially benefits the maximum possible range of the airplane. (See fig. 29.) The benefit diminishes, however, to almost a negligible amount for speeds considerably above the speed for maximum range. The almost obvious reason for this decrease is that the induced drag, the element affected by the span loading, constitutes about 50 percent of the total drag at $\left(\frac{L}{D}\right)_{\max}$ but at higher speeds the percentage is much less. The speed for $\left(\frac{L}{D}\right)_{\max}$, of course, changes with span loading. (See fig. 1.)

As might have been expected, increasing the parasite loading by decreasing the parasite drag increases the range nearly uniformly for all speeds of flight. (See fig. 30.)

It can be seen that a rather complex situation arises in proportioning wing dimensions for the best range condi-

tion, If the wing constitutes the entire airplane, increasing the effective span by increasing the wing area while retaining the same aspect ratio would not affect the L/D . As the wing ordinarily constitutes only a part of the total airplane drag, increasing the wing area would increase the span at a faster rate than the total parasite area, with a higher resulting $\left(\frac{L}{D}\right)_{\max}$. At speeds somewhat higher than the speed for $\left(\frac{L}{D}\right)_{\max}$, increasing the wing area may affect the L/D adversely because the increase in parasite drag may be greater than the decrease in induced drag.

Increasing the span without increasing the chord or thickness would materially increase the maximum range but would not greatly affect the range at relatively high speeds.

Figure 31 shows the effect of increasing both the span and parasite loadings in the same ratio. For this analysis it was assumed that the designed weight of the airplane remained constant while the size was decreased in steps to half its original value. The principal effect of such a procedure is an almost uniform increase in designed speed for a given range, while the maximum possible range was little affected. If all the factors involved remained constant except the loading, it would be expected that the normal loading curve would be displaced proportionally to the relation $\frac{V}{V_0} = \sqrt{\frac{W}{W_0}}$. Actually, the curve is displaced more than this amount because both the propulsive efficiency and the relative engine weights changed. The chief disadvantage of increasing the loading is the increased take-off run and landing speed. The take-off run for this condition will be discussed more fully.

Example 5, Composite Condition

Design and flight conditions:

Sufficient power to cruise at 0.75 rated power at the initial part of the flight.

Engines supercharged to 15,000 feet critical altitude.

1.6 normal span and parasite loadings.

Flight at constant L/D at 19,500 feet altitude.

In the preceding examples some of the important factors affecting the range were individually analyzed. The question naturally arises as to whether the effects are cumulative or whether certain factors cancel when several variables are changed at once. Figure 32 illustrates the effect of incorporating the most obvious methods for improving the designed speed of flight for the airplane. The conditions assumed are all within reach of the designer and operator at the present time. Flight at 19,500 feet would, of course, be uncomfortable for passengers and crew unless special precautions were taken to compress the air in the cabin or to supply oxygen. Operation at an altitude of 19,500 feet represents full-throttle operation at 0.75 power for the engine rated at a critical altitude of 15,000 feet. Only 4,500 additional feet were necessary at this altitude to reduce the torque the required amount as compared with an altitude of 5,000 feet for unsupercharged engines.

Figure 32 illustrates the marked improvement in the designed speed gained without a large sacrifice in range. For the conditions assumed it can hardly be said that the effects were directly cumulative for it appears that the factors reacted favorably together to produce higher speeds than were expected.

The conditions assumed for this example are purely arbitrary. They were chosen with the idea of obtaining the greatest gain in operating speed with the least loss in range. Operation at higher altitudes with superchargers of higher-altitude capacity and with airplanes of higher designed loadings would accentuate the effects illustrated, to a certain extent, but would be more difficult to accomplish. The chief difficulties with operating at higher altitudes are those of supplying oxygen or compressed air to the passengers and developing a supercharger of unusually high-altitude capacity. Both of these factors are in the experimental stage at the present time. There probably is an economic limitation in going to higher altitudes, as far as range is concerned, although this example does not necessarily represent the limit.

If, as in this example, an airplane is designed to

operate at a certain altitude and at a certain speed, the important performance characteristics at lower altitudes should be investigated to be certain that the airplane could reach the operating altitude in a reasonable length of time. In figure 33 the power curves for sea-level operation are given for several designed speeds of flight at altitude for this example. The assumption was made that the power available at sea level was the same as that for critical altitude. The curves indicate that for such conditions as were assumed it is desirable to design for relatively high speeds because of the improvement in take-off and climb possible with the greater power available. The take-off will later be more fully discussed.

Example 6, Comparison between Compression-Ignition and Spark-Ignition Engines Mounted in the Same Airplane

Design and flight conditions:

Sufficient power to cruise at 0.75 rated power at the initial part of the flight.

Normal span and parasite loadings.

Flight at constant L/D at sea level.

The merit of compression-ignition engines in lengthening the range has already been discussed in some detail. In order to illustrate more fully the effect of replacing spark-ignition with compression-ignition engines, an example is presented. In figure 34 the range at different design speeds is given for both types of engines having different assumed weights and fuel consumptions. It may be noted that in this example for relatively short flights the advantage lies with the spark-ignition engine; for long flights the reverse is true. This comparison illustrates the relative importance of specific fuel consumption and engine weights for conditions of differing amounts of fuel carried. Figure 34 further brings out the fact that the possibility of improving the range by means of compression-ignition installations diminishes with increased designed speed of flight because the larger engines required for higher speeds affect the weight of the compression-ignition engines to a greater extent than it does that of the spark-ignition engines.

It should be emphasized that the actual range at which the two types of engines are equal depends upon their relative specific weights and fuel consumptions. With improvements in compression-ignition engines the difference in specific weights may be lessened; in this case the range at which the types of engines are equal in merit will be lowered and the advantage of the compression-ignition engine for long ranges will be increased.

On the other hand, recent reports on both carbureted and fuel-injection engines indicate that the specific fuel consumption and the weight of spark-ignition engines may be substantially reduced below the value used for this example. These reductions have the effect of increasing the range in which the two types of engine are equal and making it more difficult to bring the compression ignition to a parity with the spark-ignition for short ranges.

Little is known of the operating characteristics of compression-ignition engines at altitudes, but the indications are that the power does not decrease with decreased density as fast as it does for spark-ignition engines.

III - TAKE-OFF PROBLEM OF LONG-RANGE AIRPLANES

In the preceding portion of this paper several examples were given to illustrate the effects of certain variables on range and the speed of flight. In the examples consideration was given only to these characteristics, and none to the very important problem of take-off. Unfortunately, some of the characteristics making for long range also increase the difficulties of getting off the ground. It is therefore necessary to investigate the take-off runs of the most critical examples and devise methods, if possible, for improving them.

Assumptions and Methods

The following assumptions were made:

Controllable propellers that provided constant brake

horsepower during take-off.

For supercharged engines the power available for take-off was equal to the rated power at the critical altitude.

The attitude of the airplane during take-off was such as to give the least air and ground resistance.

Ground-rolling resistance coefficient $\mu = 0.05$.

In order to take into account the increased drag during take-off due to the retractable landing gear assumed, the minimum drag coefficient of the airplane was increased 25 percent.

The speed for take-off was equal to the speed corresponding to $0.8 C_{L_{\max}}$, where $C_{L_{\max}} = 1.35$. This assumption applied for the condition of no flaps. When flaps were employed, the speed was such that a reserve thrust of 500 pounds existed beyond the requirement for level flight.

A form of graphic integration was employed for determining the length of take-off. In this method V/a was plotted against velocity, wherein

$$\frac{V}{a} = \frac{\frac{VW}{g}}{T - \left[\mu W + \frac{\rho V^2 S}{2} (C_{D_1} - \mu C_{L_1}) \right]}$$

where V is the air speed, f.p.s.

a , acceleration, ft./sec.²

g , acceleration due to gravity, ft./sec.²

T , propeller thrust, lb.

μ , coefficient of rolling friction = 0.05.

S , wing area, sq.ft.

C_{D_1} , drag coefficient in take-off attitude.

C_{L_1} , lift coefficient in take-off attitude.

The thrust at take-off may be computed by any of several methods. Since controllable propellers were assumed, it was found convenient to obtain the thrust for different velocities by using data given in reference 6. The method employed is illustrated in the following table.

SAMPLE COMPUTATION ILLUSTRATING METHOD EMPLOYED
FOR DETERMINING PROPELLER THRUST

(Controllable propeller. Diameter = 10.0 ft.
500 hp. at 1,950 r.p.m. $C_p = 0.0337$)

Air speed	$\frac{V}{nD}$	Pitch	C_T	Thrust
f.p.s.		Degrees		Pounds
20	0.062	15.0	0.085	2,130
40	.123	14.8	.081	2,030
60	.185	14.7	.076	1,905
80	.214	14.6	.070	1,755
100	.308	14.7	.065	1,630
120	.369	14.9	.059	1,490
140	.431	15.3	.055	1,380
160	.492	15.9	.051	1,280

The power coefficient C_p remaining constant, the pitch was determined for different values of V/nD corresponding to the velocity. The thrust coefficient C_T and the thrust were then obtained. In figure 35 examples of thrust curves for different take-off speeds are given.

General Take-Off Curves

In figure 36 general take-off curves are given for

the airplane assumed for this analysis. This figure illustrates the importance of propeller diameter and wing and power loadings for take-off considerations. It is unfortunate that such a simple relationship cannot be made to apply to all sizes and types of airplanes. A general relationship would require, at least, the addition of the important parameters - engine speed, weight, and parasite loading - and would be difficult to establish in a simple manner.

Take-Off Runs for Different Loading Conditions Assumed in Example 4

Without flaps.- The only condition to be considered with respect to take-off is that in which both the span and parasite loadings are changed proportionately by altering the airplane dimensions. In this assumed method of altering the span loading, the wing loading changes proportionately and the comparison for take-off runs can be put on a wing-loading rather than a span-loading basis since take-off run is more nearly a function of wing loading.

In figure 37 the take-off runs are given for different designed speeds of flight and different loading conditions. It may be noted that the take-off run was confined to reasonable values for the moderate loadings by incorporating sufficient power to enable the airplane to cruise at fairly high speeds. Extreme take-off runs were inevitable for designed conditions of flight at $\left(\frac{L}{D}\right)_{\max}$.

In figure 38 the take-off runs corresponding to different designed conditions affecting range have been plotted. If, for example, the take-off run had been limited to 3,000 feet, it would have been necessary to provide sufficient power to fly at 126 miles per hour for the normal loading or 189 miles per hour for the 1.6 normal loading. If the airplane were flown at the designed speeds, the maximum range would have been reduced about 9 percent for the normal loading or about 27 percent for 1.6 normal loading. From these results it appears that a large part of the advantage of speed and range gained by resorting to high loadings is lost by the restrictions imposed by the take-off.

Another method of attacking the problem is that of adding sufficient power for the desired take-off but flying at only the desired speed, instead of at the speed

corresponding to 75 percent rated power at the initial part of the flight. By this method range is sacrificed mostly by virtue of the increased engine weight and slightly by increased fuel consumption. In figure 39 the power required curves for certain designed range conditions are given together with curves of power required for a take-off run of 2,500 feet, which has been assumed to be the maximum allowable run. The values indicate that the additional power required for take-off increased rapidly with loading. In figure 40 the effect on range of adding power for take-off is indicated. For the lower loadings the added engine weight does not decrease the range greatly; but for the higher loadings the effect is more pronounced.

Effect of flaps.— In view of the higher lift coefficients and correspondingly lower minimum speeds possible with flaps it appears that they might be beneficial in decreasing the take-off run for heavily loaded airplanes. A study of the subject (reference 12) indicates that the Fowler flap ranks among the best of the types having possibilities for improving the take-off. It therefore appeared desirable to determine the merits of such flaps for airplanes designed for long range. Full-span flaps with a chord equal to 30 percent of the main wing chord depressed 30° were assumed.

In figure 41 drag curves for the airplane are given for the conditions of flaps up and down. Thrust curves are also included for several engine-propeller combinations. It may be noted that if sufficient thrust were available the take-off could be accomplished at a much lower speed with the flap than without. On the other hand, if only a small thrust were available take-off could not be accomplished with the flaps but could be without them as indicated by the thrust curve for 300 horsepower. Inasmuch as the speed for take-off is more important than drag, it appears that, for the conditions wherein flaps can be used, there will be a definite benefit. In figure 42 the effect of flaps on take-off run is illustrated for a condition wherein there is sufficient thrust for a comfortable margin for taking off with the flaps down. This diagram shows that, even though the added drag due to the flaps accounts for poorer acceleration, the take-off run is greatly reduced because of the lower speed for take-off. Figure 43 further shows that the take-off run could be greatly reduced with flaps if sufficient power were available but, for the extreme low-power condition, that the flaps would have been a disadvantage. Comparative

take-off runs are given for differently designed speeds of flight and loadings in figure 44. For the lower loadings the flaps were beneficial only for the relatively high designed speeds but, for the highest loadings, the flaps were beneficial for all designed speeds investigated. This effect is brought out more fully in figure 45 wherein take-off runs are plotted for different conditions of range and speed of flight. A comparison with figure 38 indicates that flaps are beneficial only for high loadings and high designed speeds.

If sufficient power were added to accomplish a take-off within 2,500 feet, the range would be decreased less with flaps than without, as can be seen by comparing figure 46 with figure 39 and figure 47 with figure 40. These figures show that the additional power required for the assumed take-off run was much less with flaps than without, especially for the high loading conditions. The resulting lower engine weights with flaps resulted in greater maximum range. (See fig. 47.)

It therefore appears that the range limitations imposed by the effect of high loadings on take-off is slight if oversize engines and flaps are employed, less than if flaps were not employed. Had account been taken of the weight of the flaps, the results of the analyses would have been slightly modified.

Take-Off Runs for Airplanes Designed for Altitude Flight

Unsupercharged engines.— The design conditions that affect the thrust power available at take-off or the speed for take-off will affect the take-off run. For the conditions wherein oversize engines are installed for the purpose of flying unsupercharged at altitudes or for flying at high speeds, the take-off run will be materially reduced.

Supercharged engines.— If supercharged engines are employed for altitude flight, the power available for take-off may be assumed to be equal to the power at critical altitude, provided that controllable propellers are used and that the engine does not detonate. If the airplane is designed to fly at the same L/D at altitude as at sea level, the power required will be greater for altitude flight because of the higher speed. Consequently, the take-off run would be less for airplanes designed to oper-

ate at altitudes with a supercharger than for airplanes designed to operate at the same L/D at sea level and the range will not be limited by virtue of the high altitude design feature.

Take-Off Runs for Airplane Designed for Composite Condition

A take-off curve for the composite condition illustrated in example 5 is given in figure 48. It would have been necessary to design for a cruising speed of at least 222 miles per hour in order that the take-off could have been accomplished in 2,500 feet. A glance at figure 32 shows that with this speed the maximum range was reduced about 14 percent by the take-off limitations. It has been shown that flaps would materially aid in the take-off for the lower speeds, as was illustrated in figure 44 for the high loading condition. It may be necessary, also, to add power for the lower designed speeds, which would slightly decrease the range.

Aids for Take-Off Independent of the Airplane Design

In nearly all long-range flights the actual range is determined to a large extent by the amount of fuel that can be lifted off the ground within the space available. A number of schemes have been proposed to overcome this handicap; for example, fueling from another airplane after take-off has been tried with varied success. A certain amount of danger and trouble involved limits the application. Another method recently advocated abroad is one in which the long-range airplane is bodily carried aloft by means of a larger airplane and launched at a relatively high speed. This scheme also entails a certain amount of danger and complication.

It is obviously desirable that the airplane make a normal take-off with full load without adding complications. If take-offs were made only when a strong wind were blowing the problem would be lessened, for wind has about the same effect as decreasing the wing loading. It can be seen from the illustration in figure 49 that the reason wind is so beneficial is that it eliminates the necessity for accelerating up to the ground speed for take-off in still air. As the acceleration at the latter part

of the take-off is relatively slow, because of the small excess thrust, the actual ground run saved by a wind is relatively large. Of course, wind increases the drag and decreases the propeller thrust and thereby decreases the acceleration somewhat for the first part of the take-off.

Unfortunately, wind cannot be relied upon except for shipboard take-off wherein a relative wind is present because of the movement of the ship. For ground take-offs certain auxiliary means could be devised that would augment the accelerating thrust during take-off. An auxiliary thrust could be produced by any of several methods. For example, an incline of moderate slope will materially decrease the take-off run of heavily loaded airplanes. For the illustrating example of figure 50 a 3° incline reduces the run by nearly one half. It is important that the incline extend through the latter part of the take-off run because it is there that the greatest benefit is to be had. Short inclines at the initial part of the take-off are of little value, as can be seen from any of the take-off diagrams, such as figure 49.

Another method, which would act in the same manner, is by means of a low-acceleration catapult. A winch-drawn tow cable extending across the field to the airplane would probably be satisfactory. The cable would be released at the airplane soon after take-off. In figure 50, the catapult thrust required for various runs is given. A thrust of 1,000 pounds, equivalent to 218 horsepower at take-off speed, was required to reduce the run to about one half the original length. This method appears to have the greatest possibilities of any method suggested to date for shortening the take-off run of long-range airplanes.

CONCLUDING REMARKS

This study, which is based upon certain reasonable assumptions, indicates the following general conclusions:

1. A large span is desirable for airplanes intended for flight at speeds approximating the speed for $\left(\frac{L}{D}\right)_{\max}$ but less so for airplanes cruising at high speeds, because the probable added accompanying parasite area may more than offset the small effect of the large span at high speed.

2. Controllable propellers are desirable because of the improvement in take-off and climb up to the cruising altitude but, apart from this flight condition, there is little advantage. There may be a slight advantage in changing the pitch during cruising flight in order to maintain the maximum value of propulsive-efficiency/fuel-consumption ratio. For cruising, the propeller should be selected to absorb the power at the highest allowable value of engine torque for continuous operation in order that the specific fuel consumption remain low.

3. Broadly speaking, the airplane weight, less fuel weight, is more important than drag. Hence, it may be advantageous to add parasite drag in some cases if the weight is substantially reduced thereby. Some of the largest airplanes built today are also the most efficient structurally, which suggests that large airplanes are better suited for long flights than are the small ones. The economic limit in size is not evident at the present stage of development.

4. Contrary to usual belief, neglecting take-off run, flight at either constant speed or constant power may result in longer range for a given average speed than flight at constant L/D , provided that the designed initial speed of flight is somewhat less than for flight at constant L/D .

5. It is desirable, because of the increased cruising speed, to fly at such an altitude that the decreased air density reduces the engine power to the desired cruising value. For flight at either constant speed or constant power the cruising altitude would remain nearly constant, but for flight at constant L/D the cruising altitude should be increased as the fuel diminishes in order to get the greatest benefit.

6. Neglecting take-off considerations, there is no advantage from the standpoint of range in increasing the size of unsupercharged engines in order to fly at higher altitudes. If the engines must be increased in size in order to improve the take-off, little if any range will be sacrificed thereby if the engines are effectively throttled the desired amount by flying at the appropriate altitude.

7. Cruising speeds may be greatly increased by flying at altitudes with supercharged engines, incurring only small losses in maximum range.

8. It is highly desirable from both the standpoints of speed and maximum range to increase the design loading (or gross weight for a given size airplane) to the highest possible value consistent with take-off limitations. Increasing the loading soon increases the take-off run to prohibitive values. If sufficient engine power is incorporated to enable reasonable take-off runs (2,500 feet), the maximum range will be decreased slightly with increased loadings, by virtue of the larger engines.

9. Certain types of flap will materially reduce the take-off run for relatively high loading conditions provided that sufficient power is available and to a less extent for relatively low loadings. Less range is sacrificed by adding power to secure reasonable take-off runs with flaps than without.

10. Airplanes intended to fly with supercharged engines at altitudes should have no greater difficulty in taking off than airplanes designed to fly at the same L/D at sea level provided that the engine-power output is not limited too much at sea level because of the supercharger.

11. Moderately inclined runways greatly aid the take-off particularly if they extend over the last part of the run. Perhaps an easier means of obtaining the same effect would be by providing a winch-drawn tow cable to augment the propeller thrust for the duration of the take-off.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., July 26, 1935.

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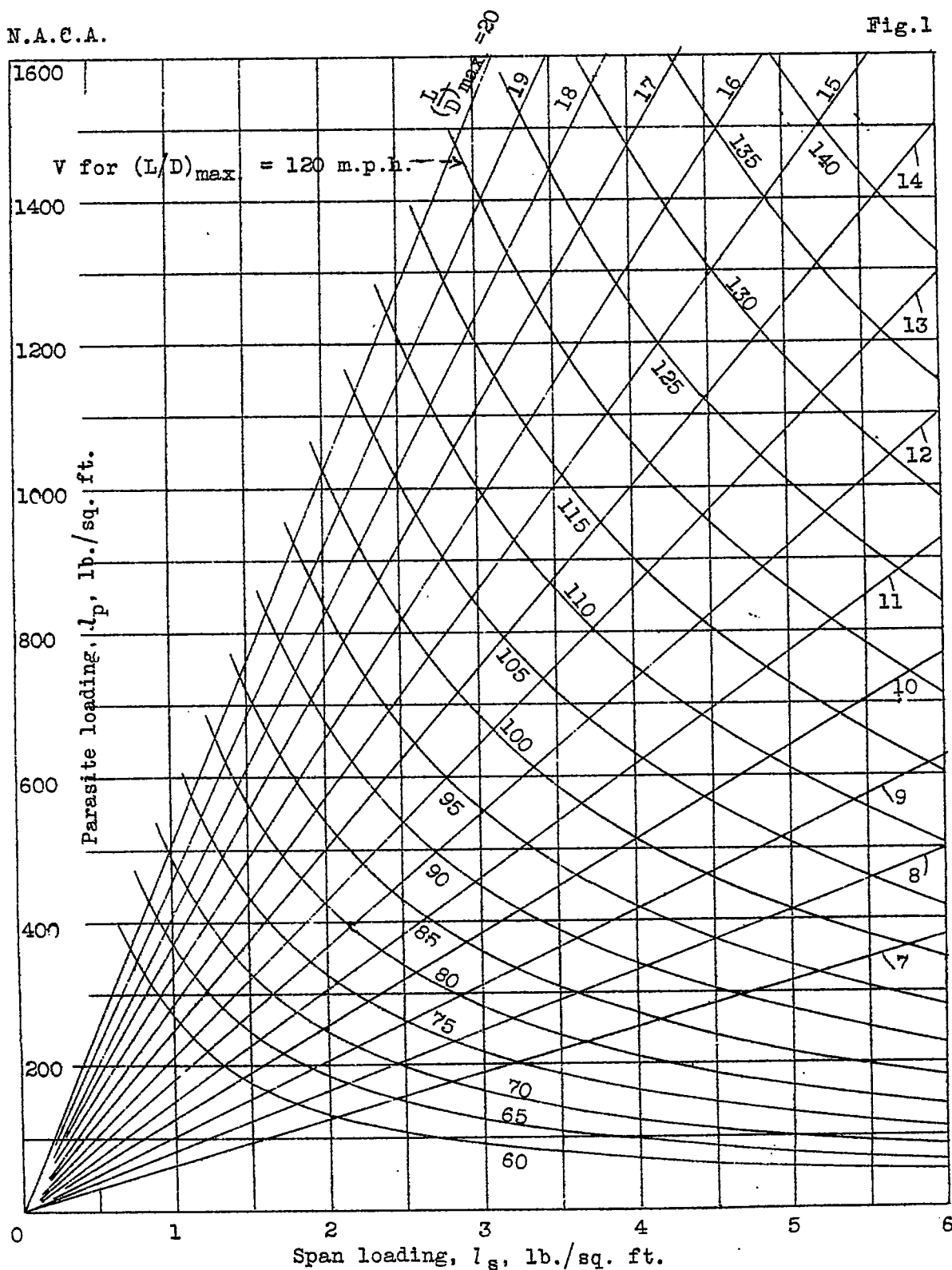


Figure 1.- The $(L/D)_{\max}$ and the velocity for $(L/D)_{\max}$ for different values of span loading and parasite loading. Sea level.

Fig. 2

N.A.C.A.

O	=	Low wing cantilever monoplane,	single engine	transport
Δ	=	" " " " " "	two " "	" "
□	=	" " " " " "	" " "	bomber
⊥	=	" " braced " "	three " "	transport
⊥	=	High " cantilever " "	two " "	" "
X	=	" " braced " "	single " "	" "
▽	=	" " " " " "	two " "	bomber
▽	=	" " " " " "	four " "	transport boat
⊥	=	Biplane " "	two " "	" "
⊥	=	" " " " " "	four " "	boat
+	=	Special cantilever monoplane,	single " "	long range
⊥	=	" " " " " "	two " "	" "

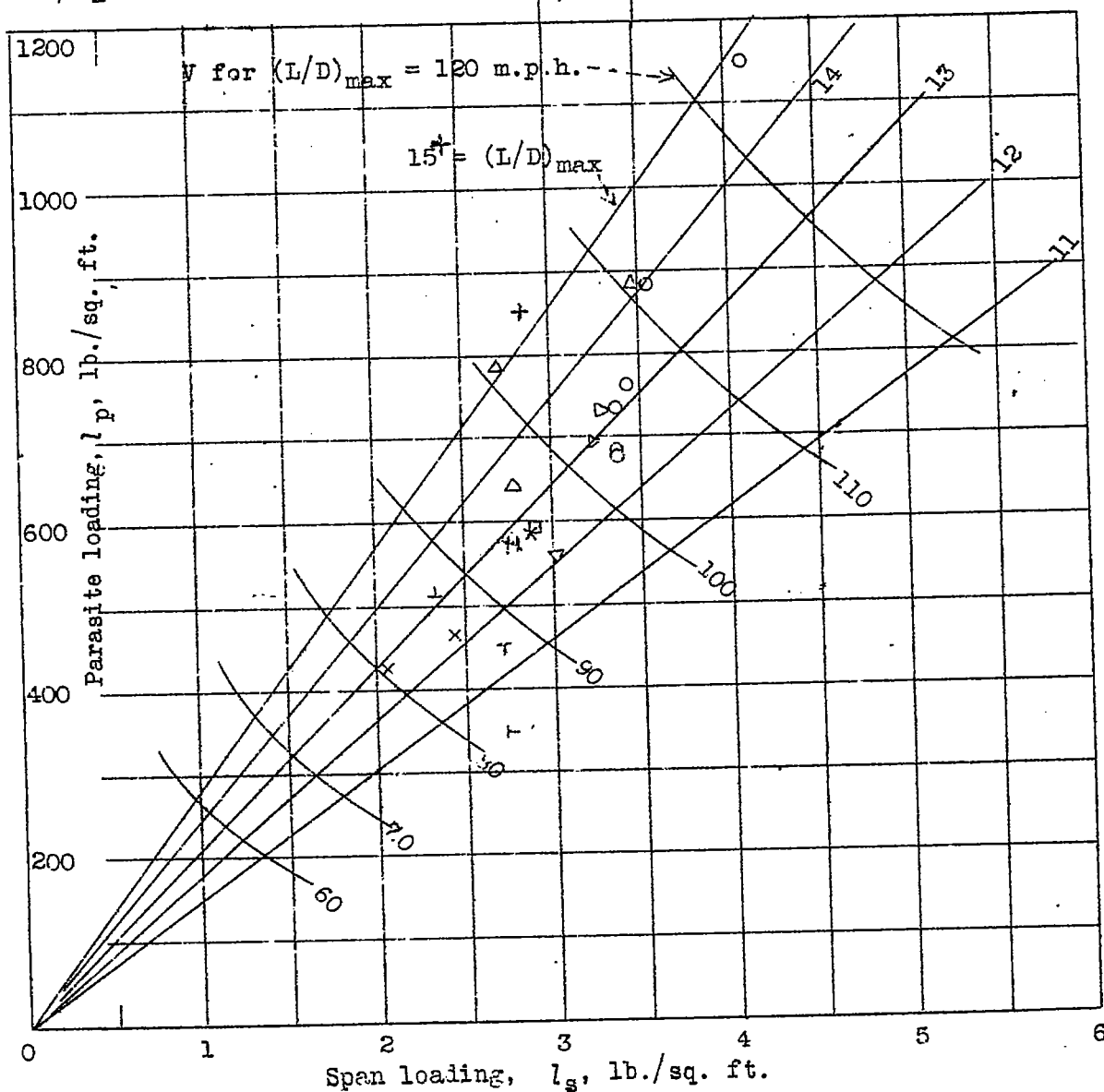


Figure 2.- Span loading and parasite loading for several modern transport airplanes.

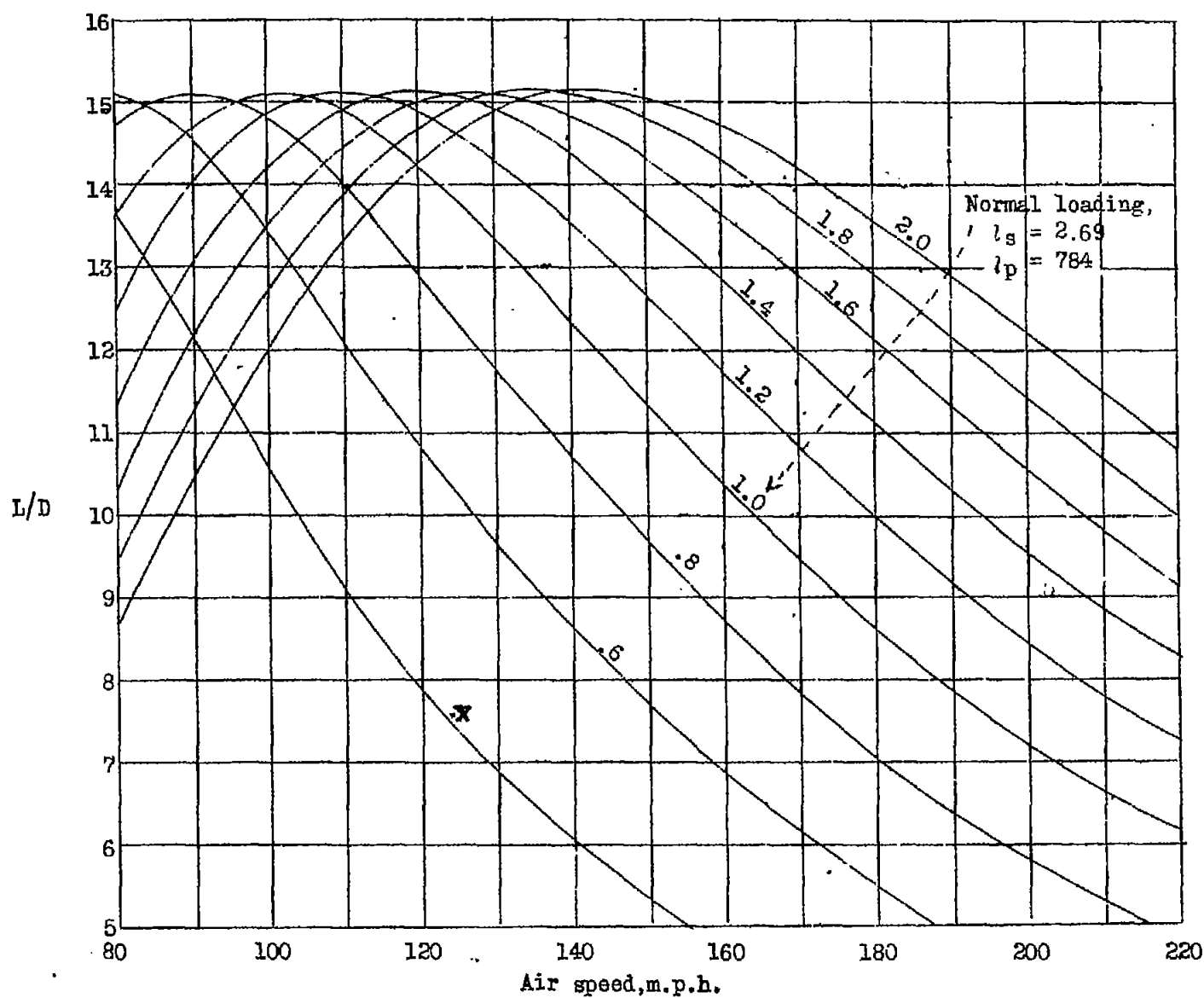


Figure 3.- Variation of L/D with air speed for different loadings for a modern transport airplane.

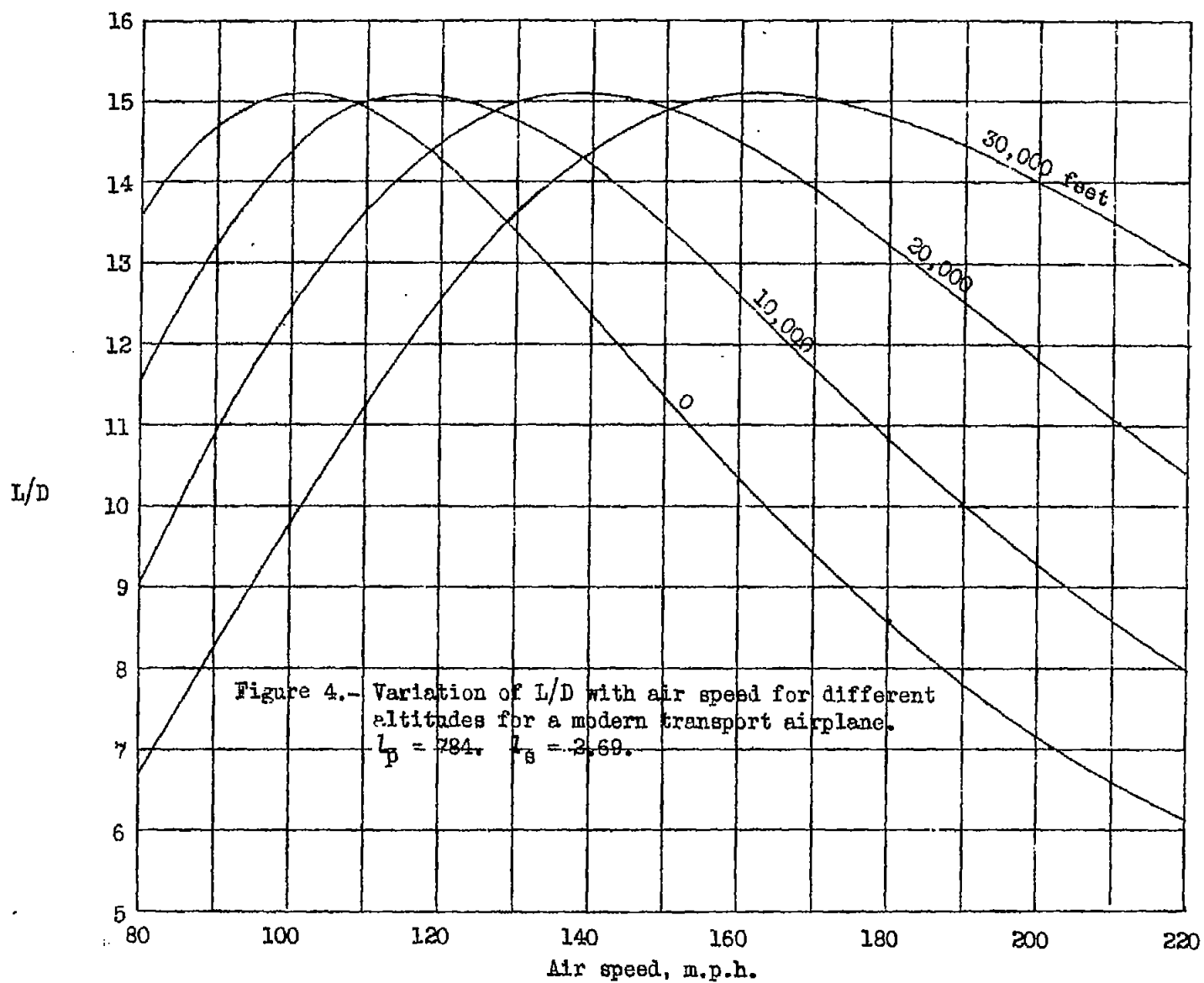


Fig. 4

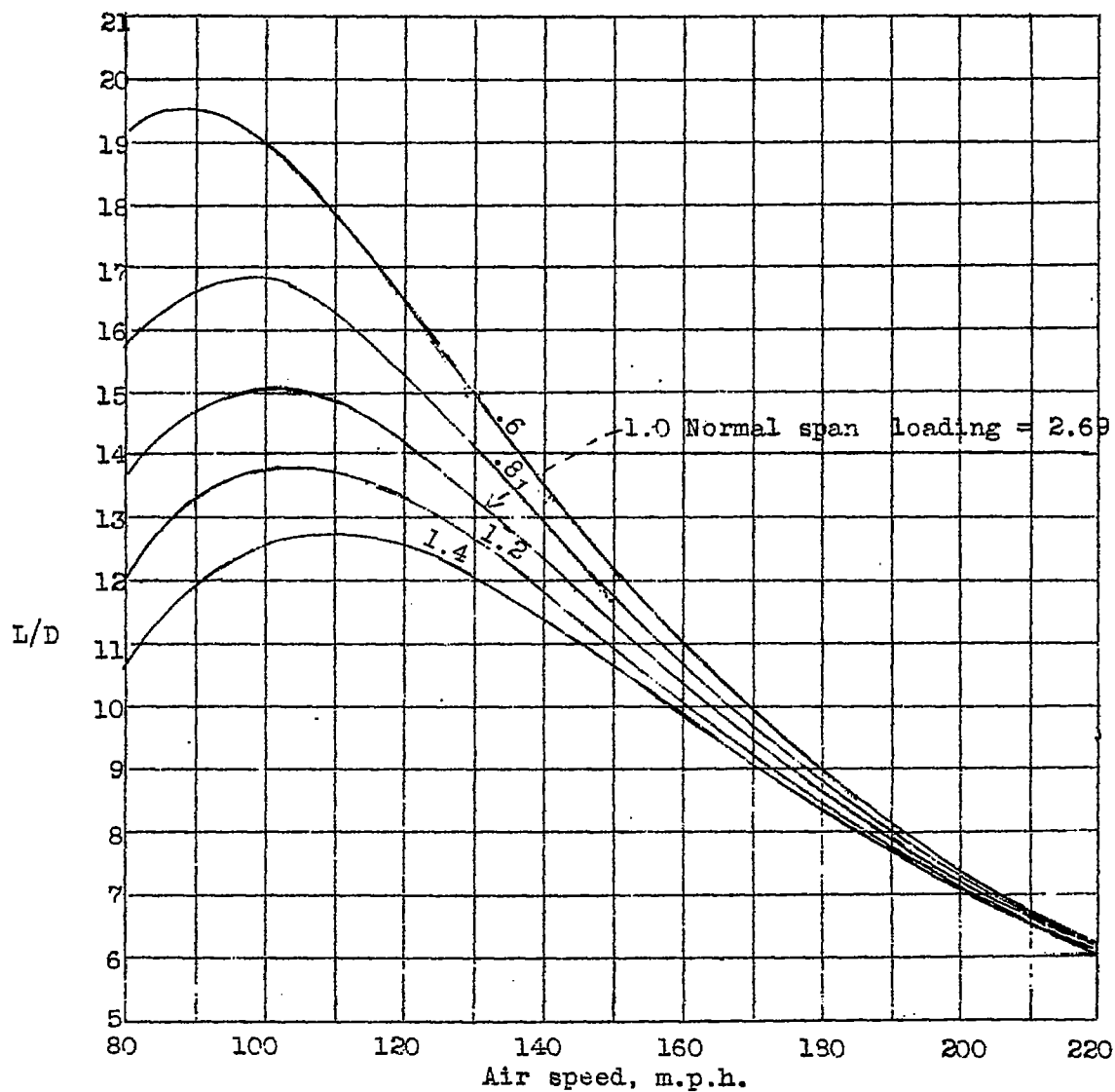


Figure 5.- Variation of L/D with air speed for different span loadings for a modern transport airplane. $l_p = 784$.

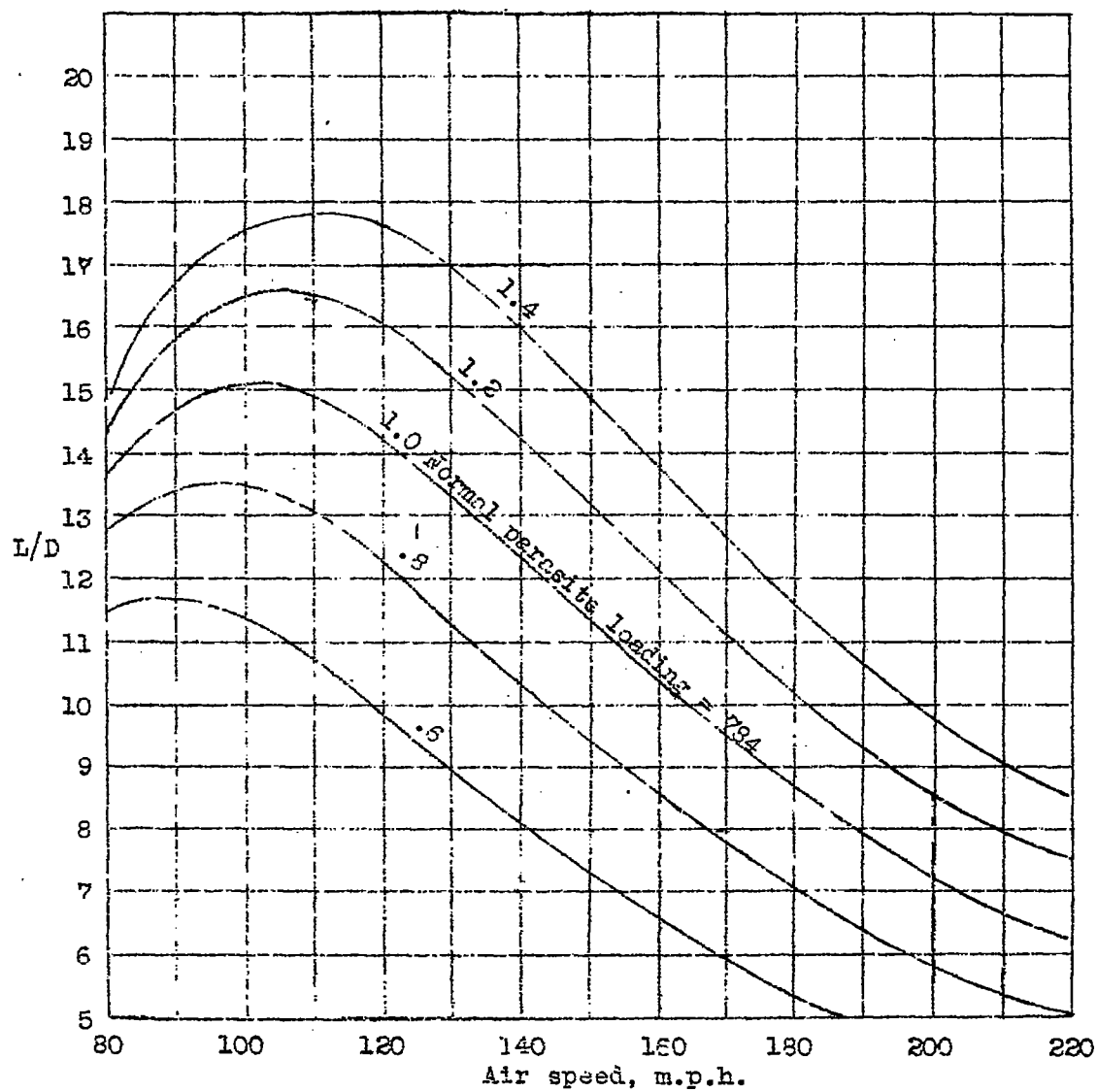


Figure 6.-- Variation of L/D with air speed for different parasite loadings for a modern transport airplane.
 $l_s = 2.69$.

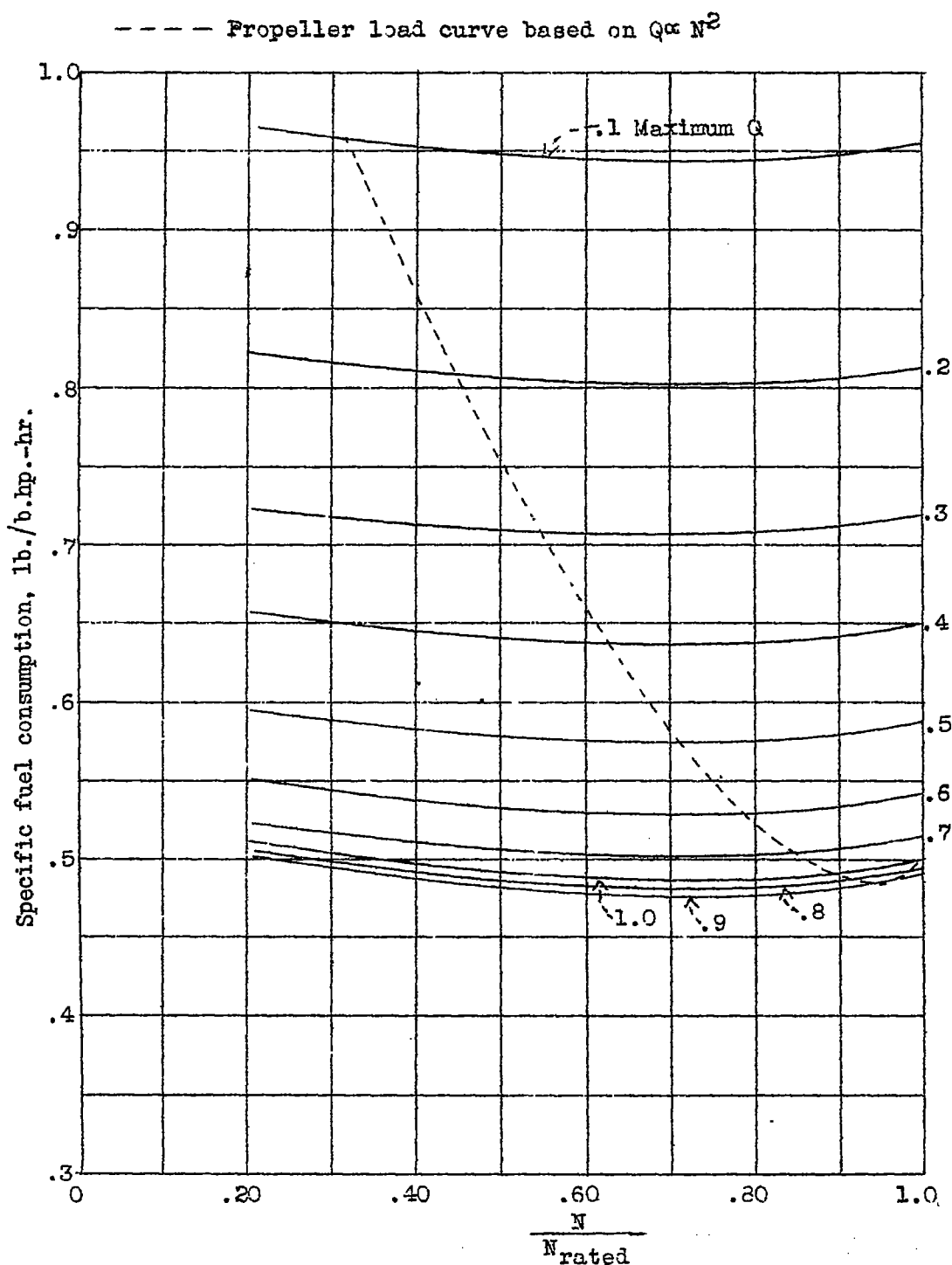


Figure 7.- Fuel-consumption curves taken from the average of a number of unsupercharged engines operating at best economy. Average compression ratio = 5.7. (The throttled torque curves have been assumed to be parallel to the full-throttle curve.)

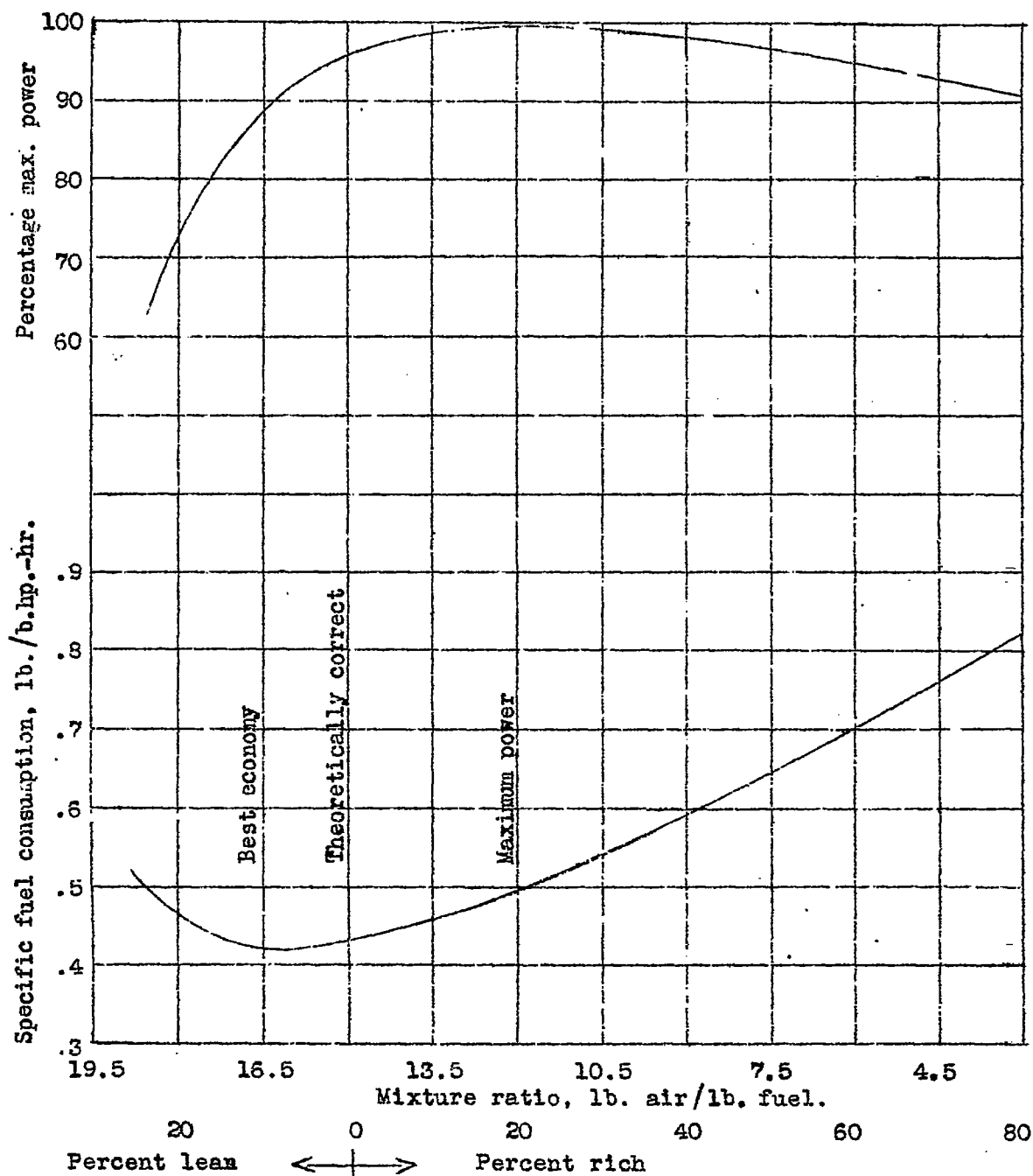


Figure 8.- Fuel consumption and percentage maximum power for different mixture ratios (from reference 7).

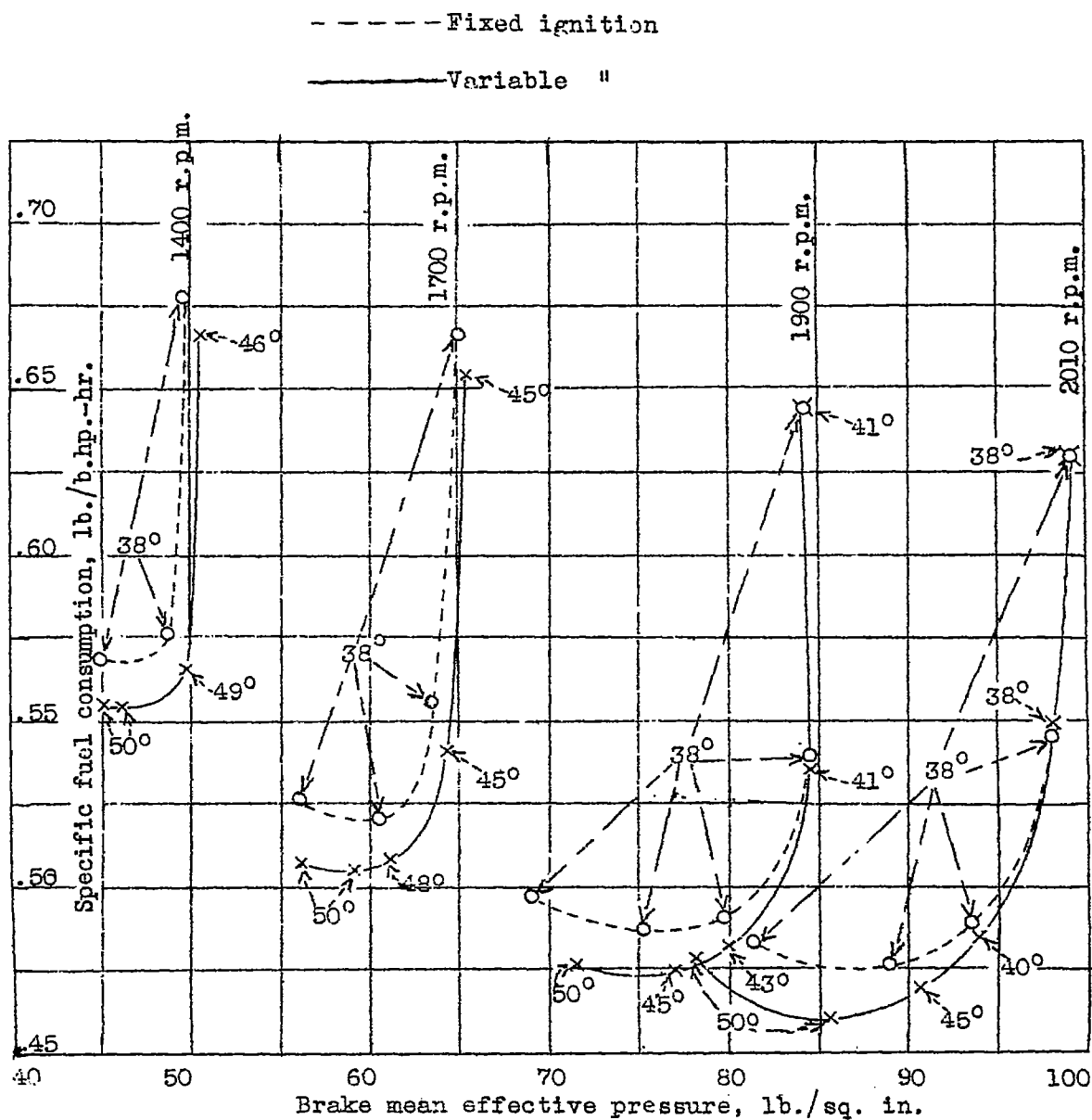


Figure 9.— Fuel consumption loops for various throttle positions with variable ignition (from reference 3).

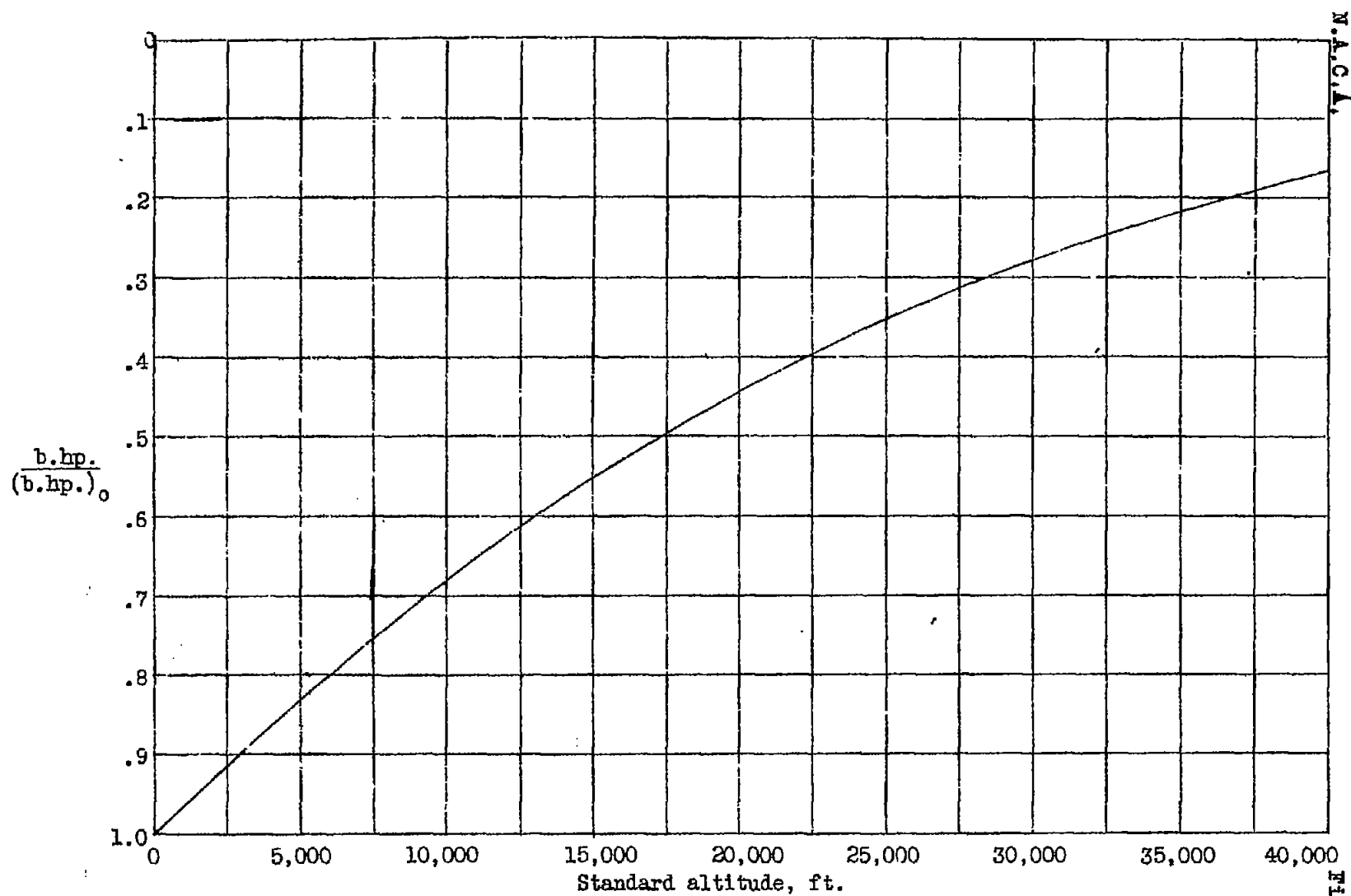


Figure 1.0.- Variation of engine power with altitude for unsupercharged engines at constant engine speed.

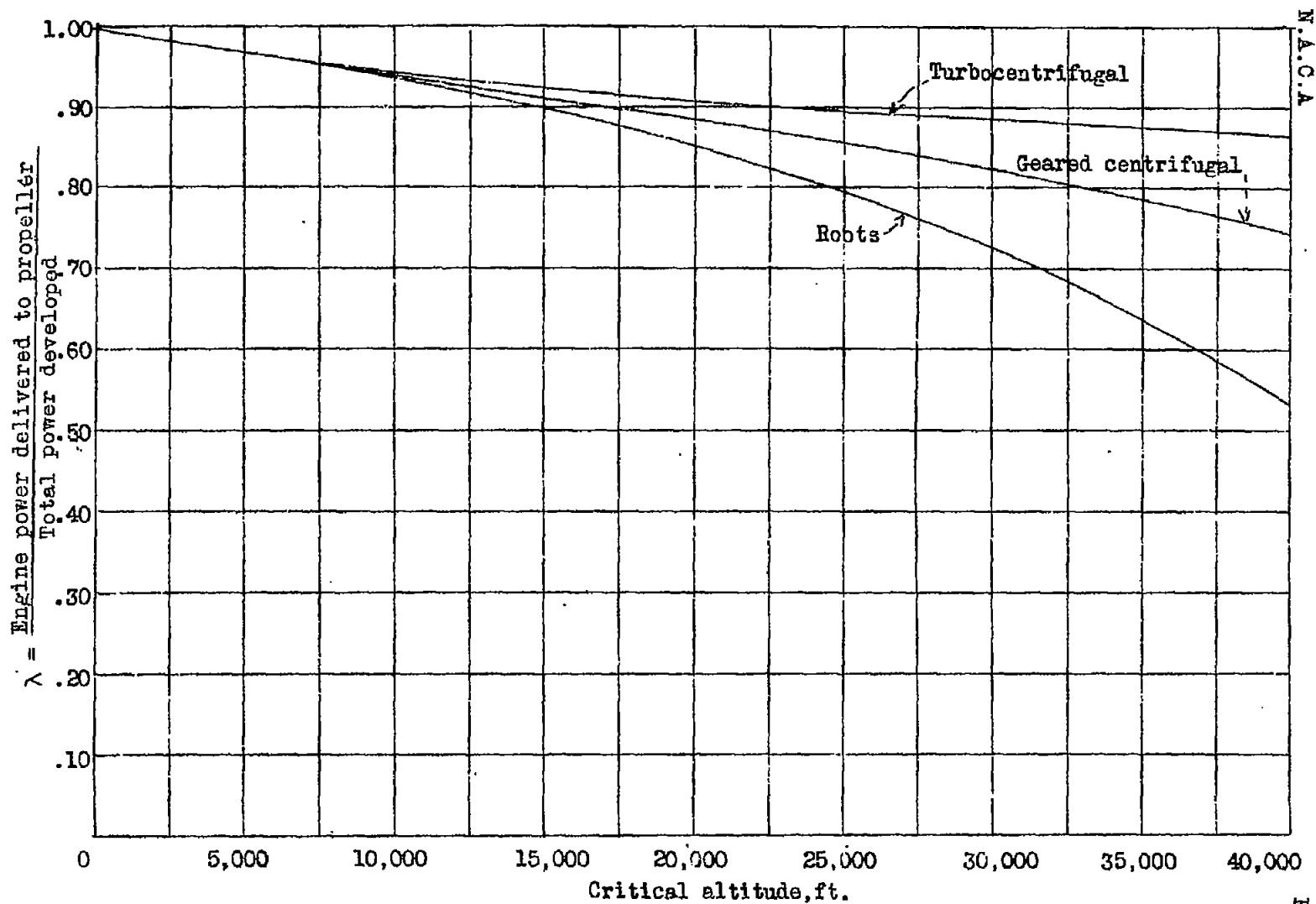


Figure 11.— Percentage of total engine power available for propulsion with different types of superchargers designed for different altitudes.

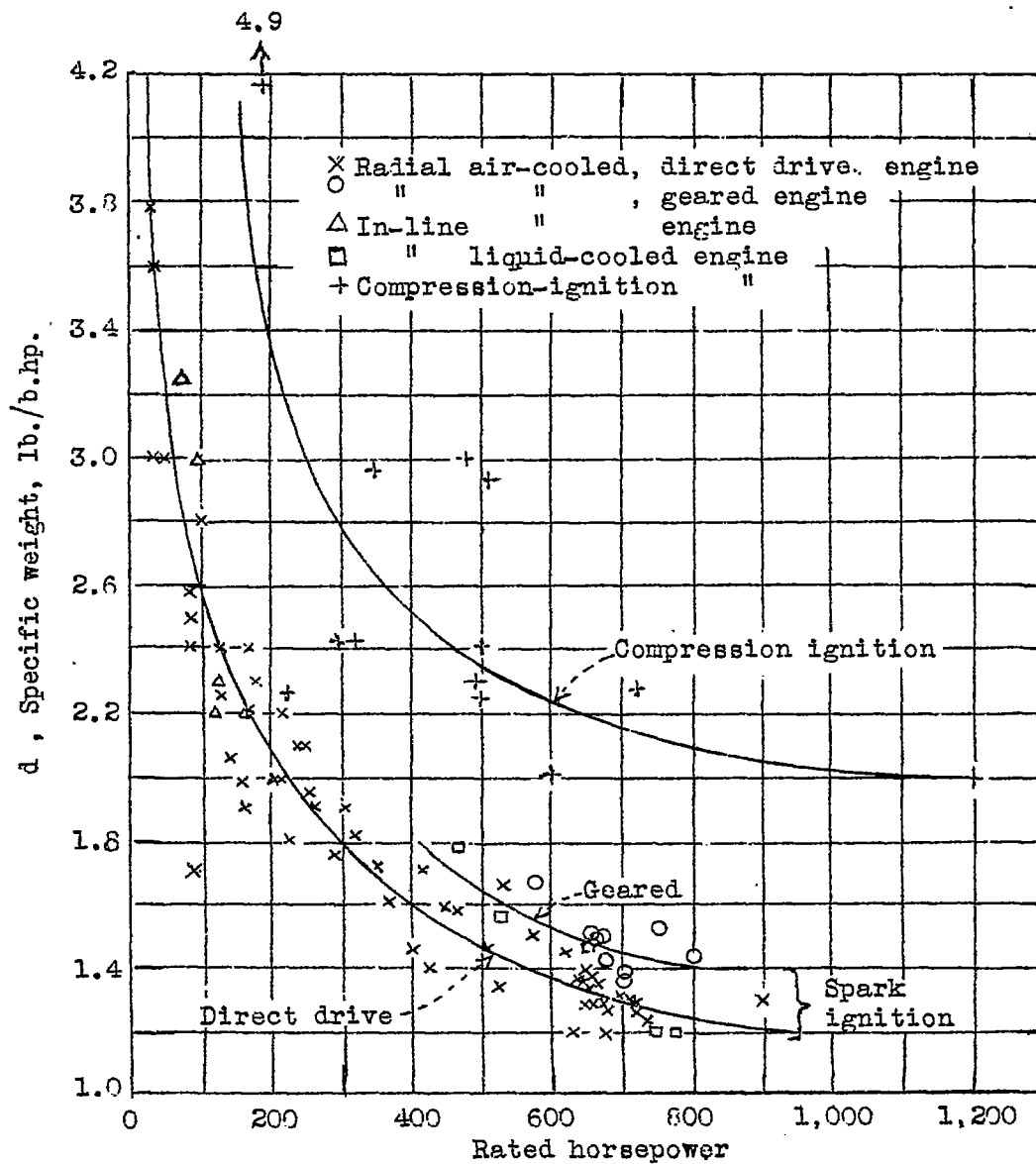


Figure 12.- Specific dry weights of modern aircraft engines.
 Estimated weight of propeller and accessories,
 0.35 lb./b.hp., estimated weight of radiator and water,
 0.58 lb./b.hp.

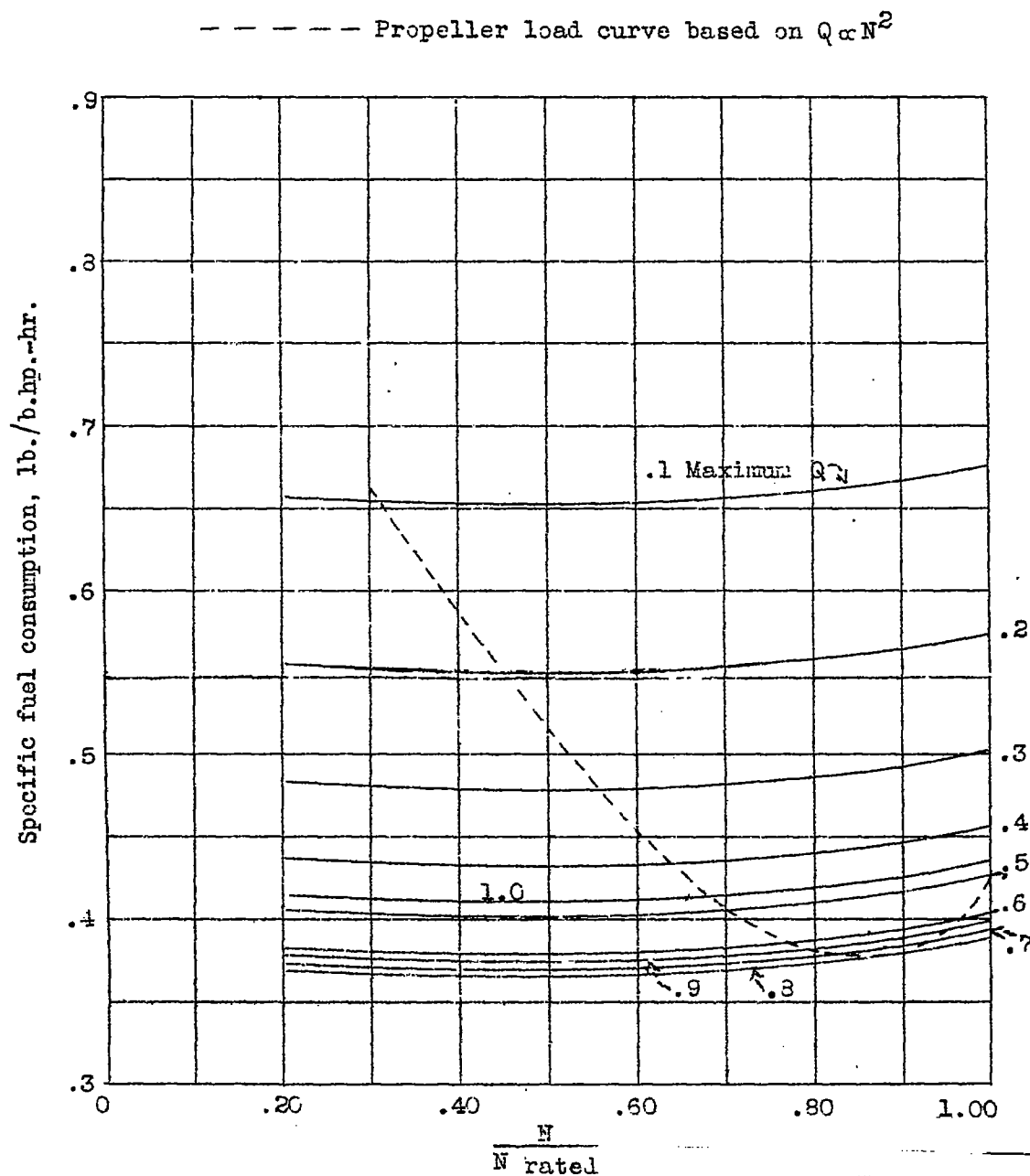


Figure 13.— Fuel-consumption curves taken from the average of a number of compression-ignition engines. (The throttled torque curves have been assumed to be parallel to the full-throttle curve.)

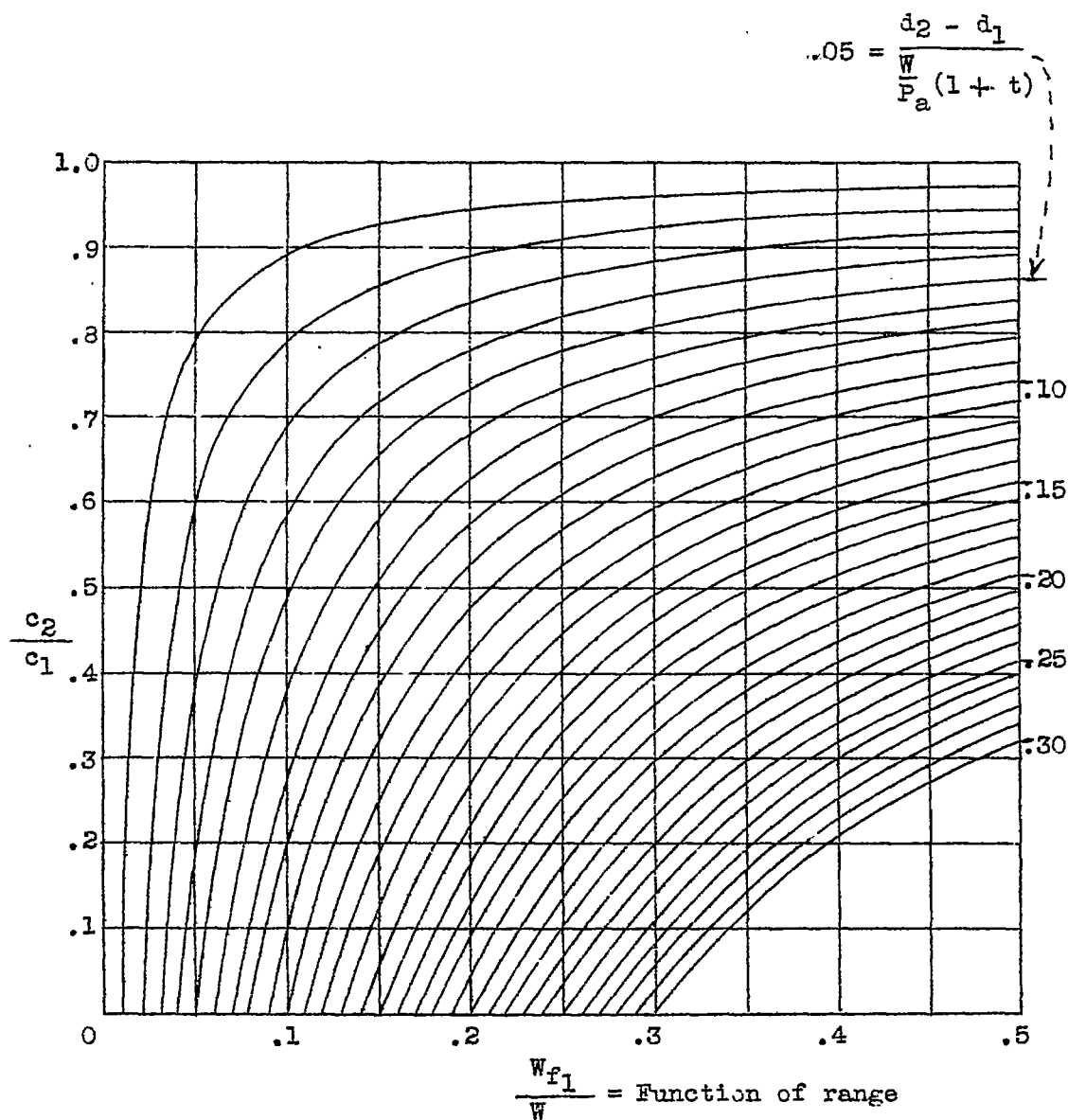


Figure 14.- A balance between specific fuel consumption and specific engine weight for two airplanes having different engine installations but of equal gross weight and range (from reference 11).

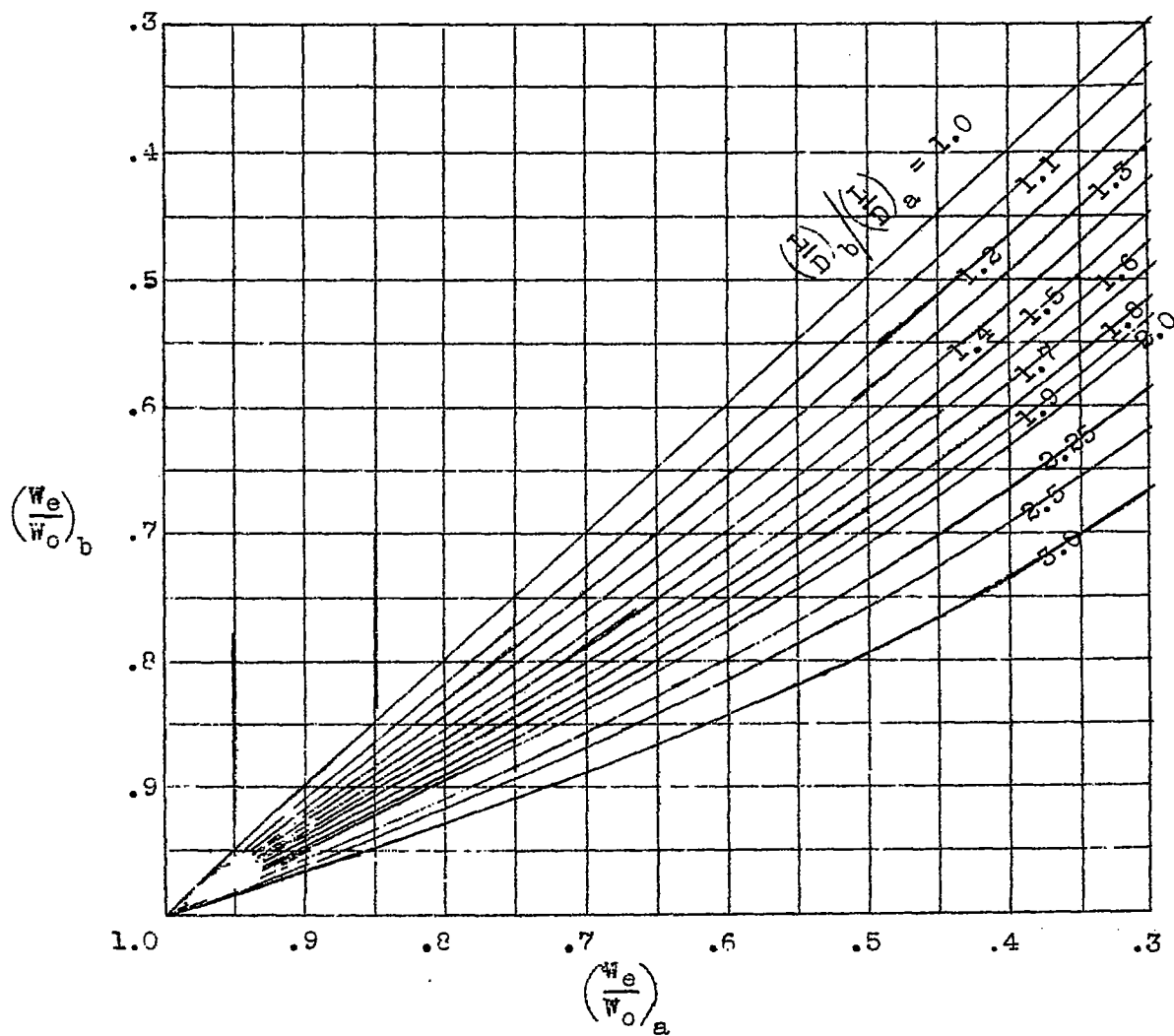


Figure 15.- Balance between the weight ratio W_e/W_o and L/D for two airplanes of equal range.

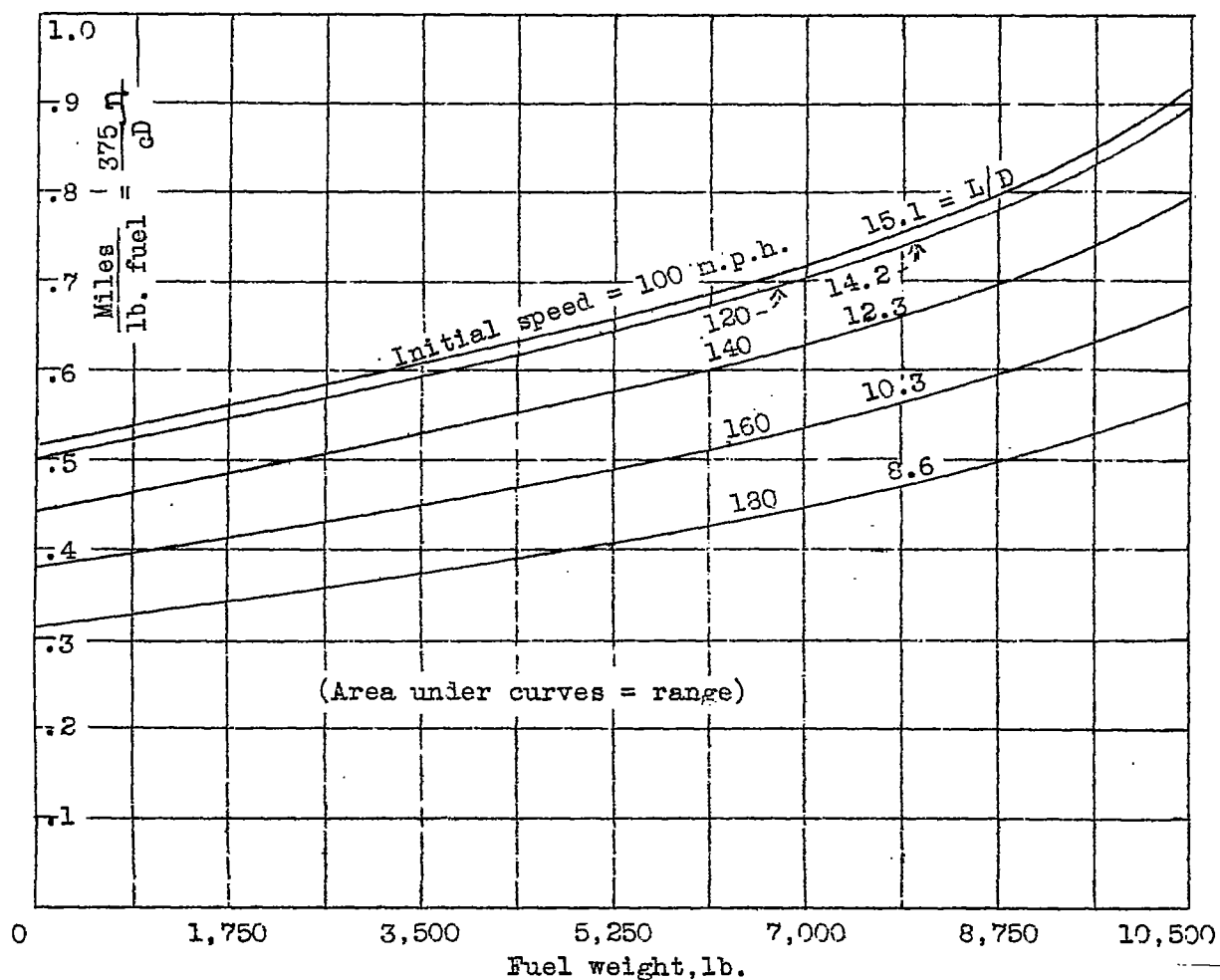


Figure 16.- Sample curves showing method used for determining the range for different fuel loads.

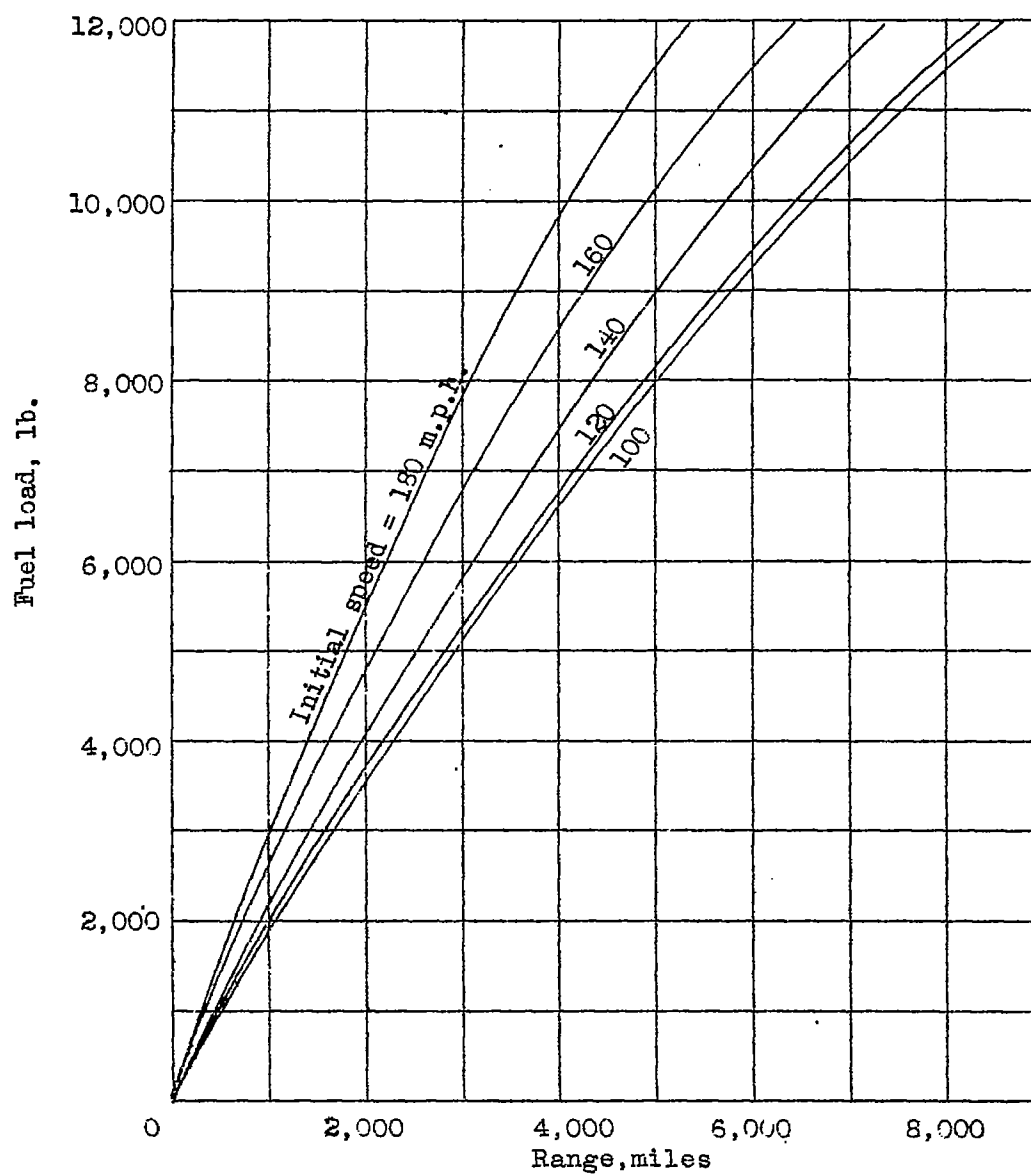


Figure 17.- Sample curves showing fuel required for different ranges when flying at different initial speeds. (Flight at constant L/D.)

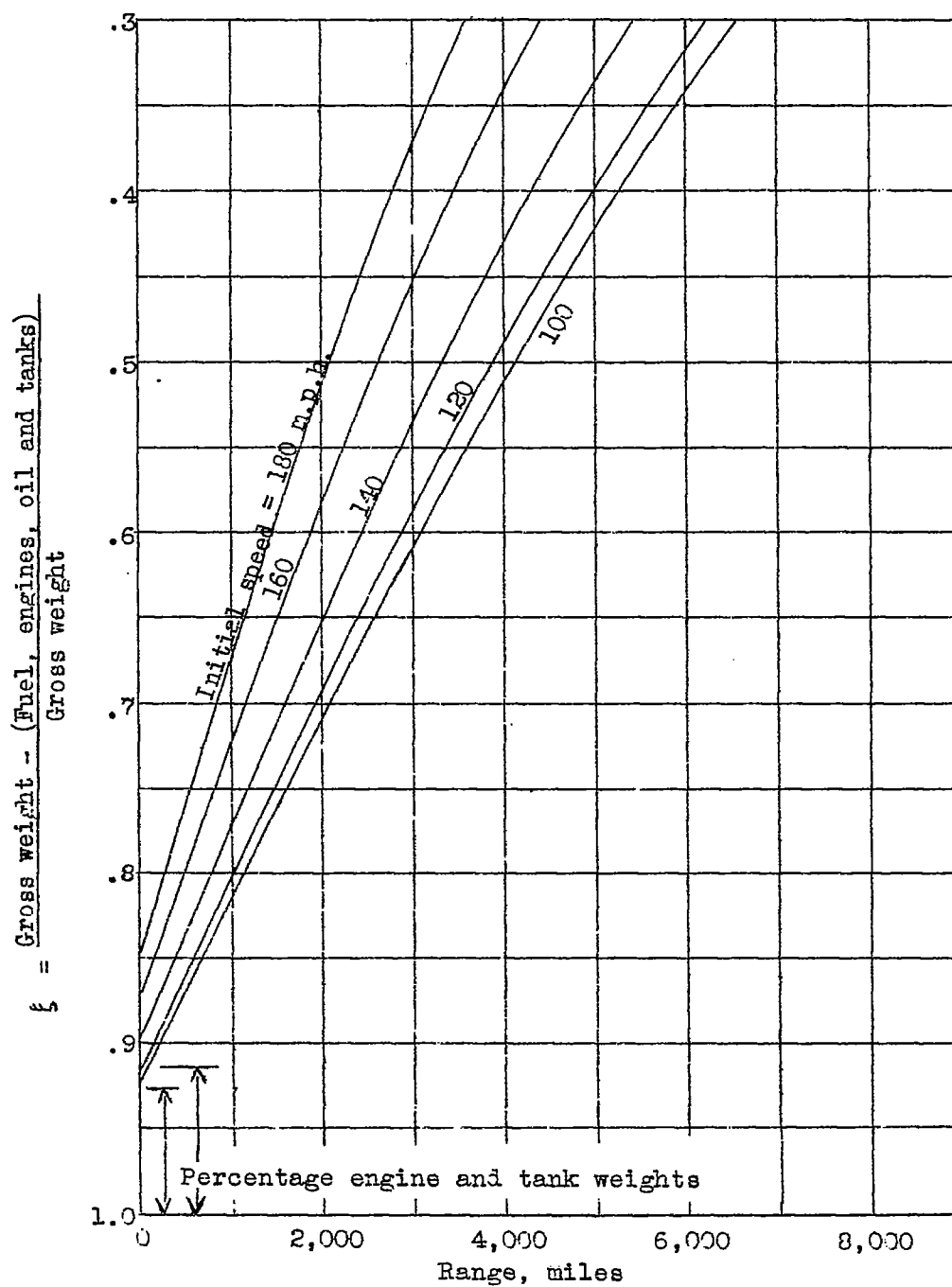


Figure 18.-- Sample curves showing the relation between ξ and range for different speeds of flight. (Flight at constant L/D.)

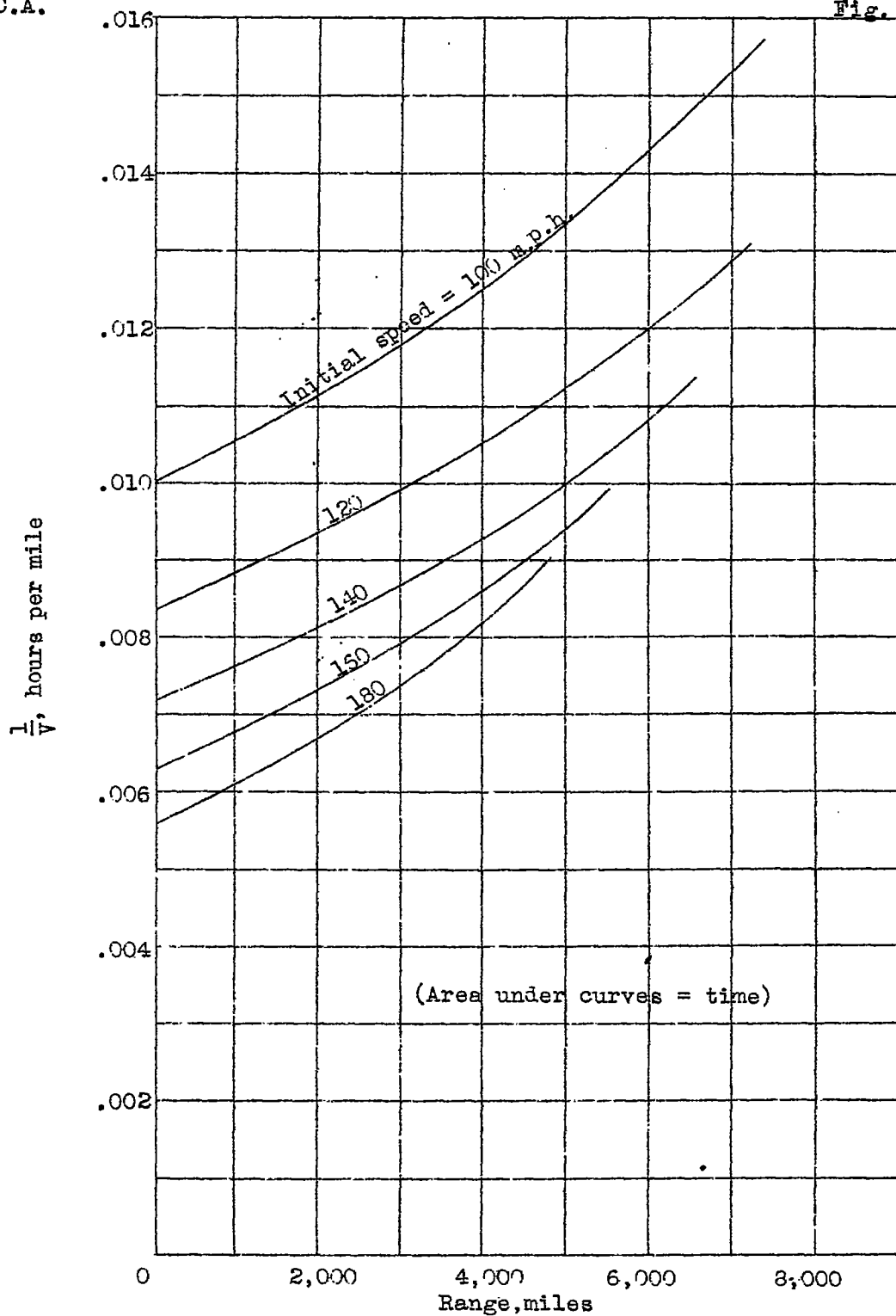


Figure 19.- Sample curves showing method used for determining the time to fly various distances at different speeds of flight. (Flight at constant L/D.)

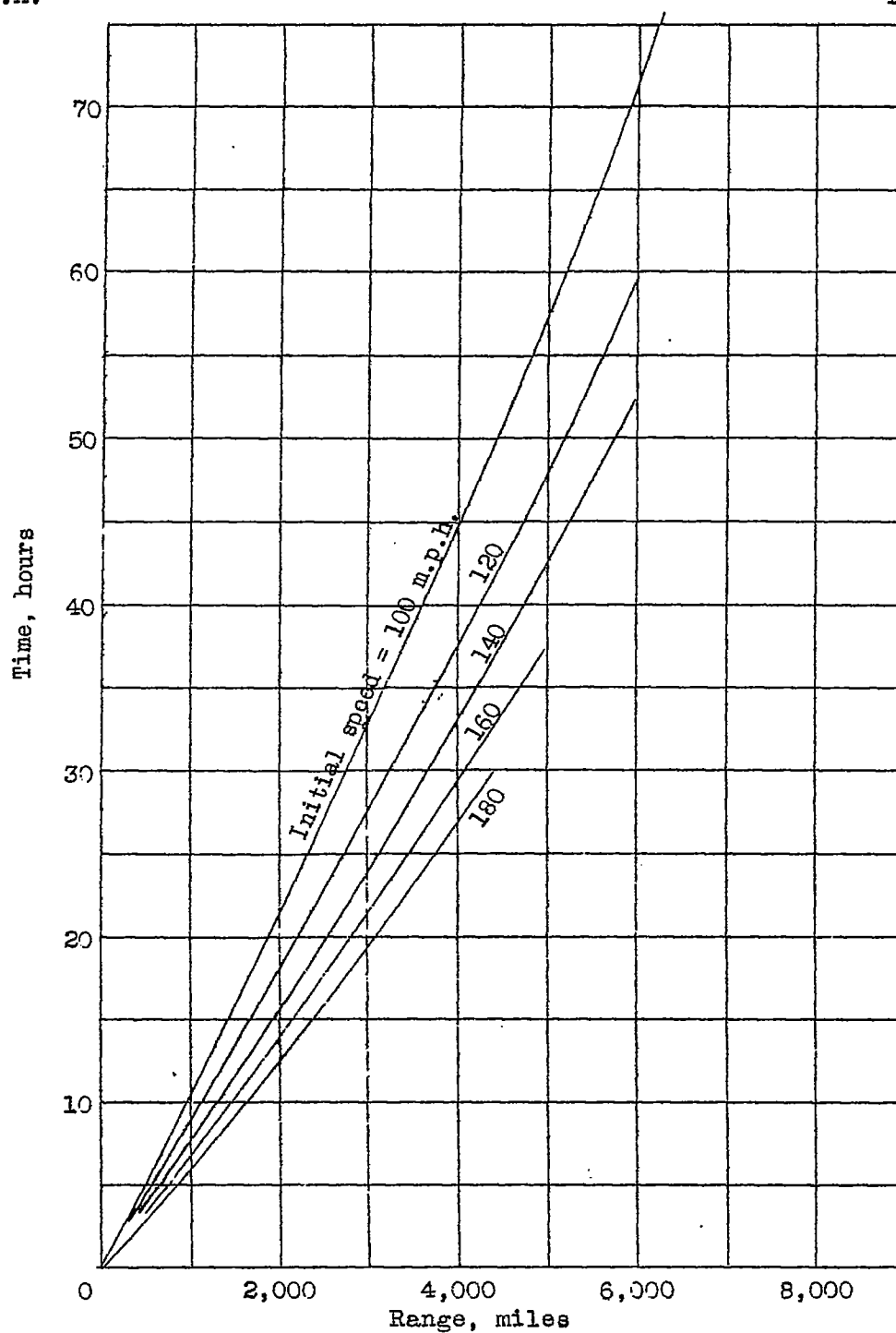


Figure 20.- Sample curves showing time required to fly various distances when flying at various initial speeds. (Flight at constant L/D .)

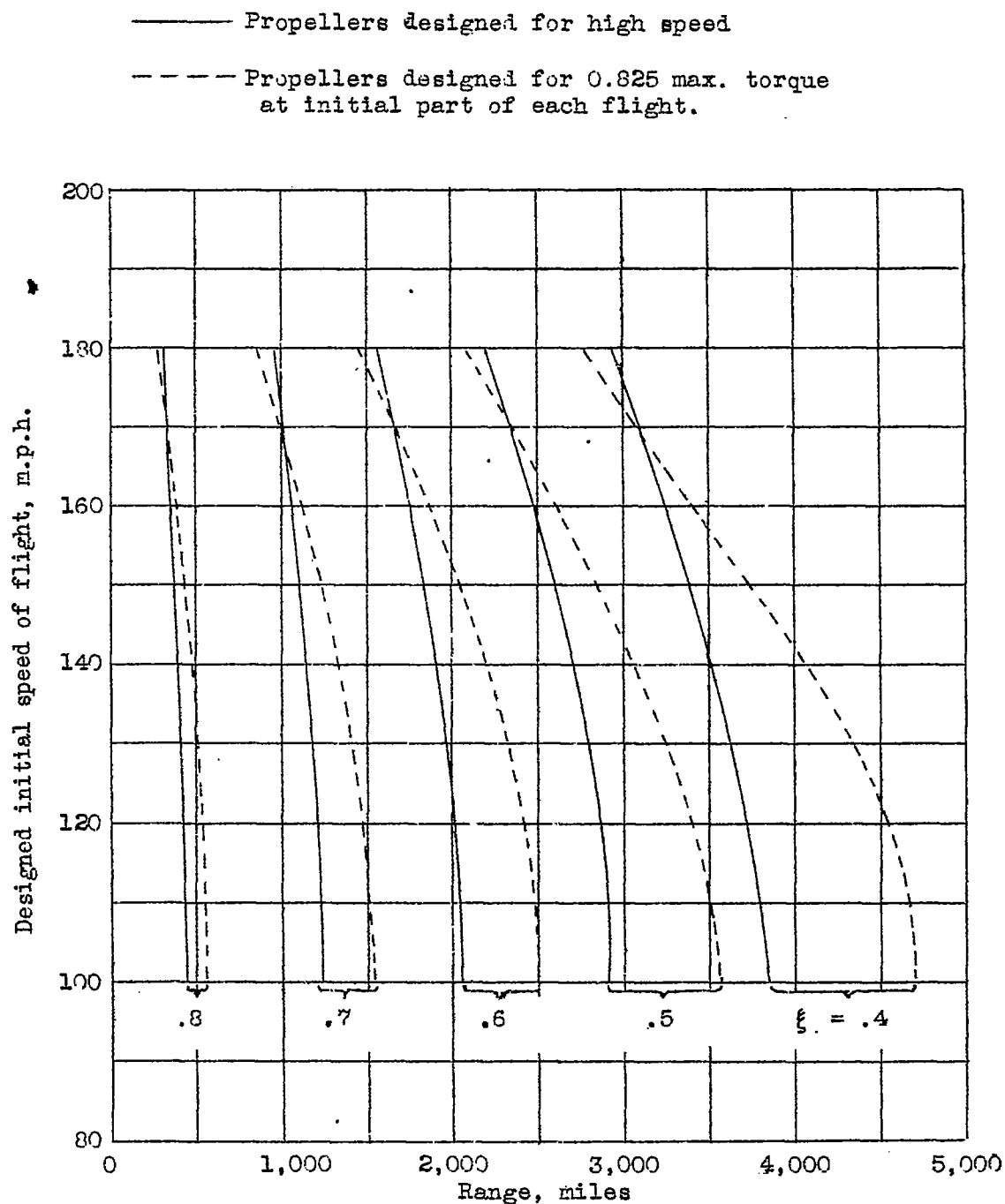


Figure 21.- Effect on range of selecting propellers for high speed, or high engine torque, for an airplane equipped with engines of sufficient power to obtain a high speed of about 200 m.p.h.

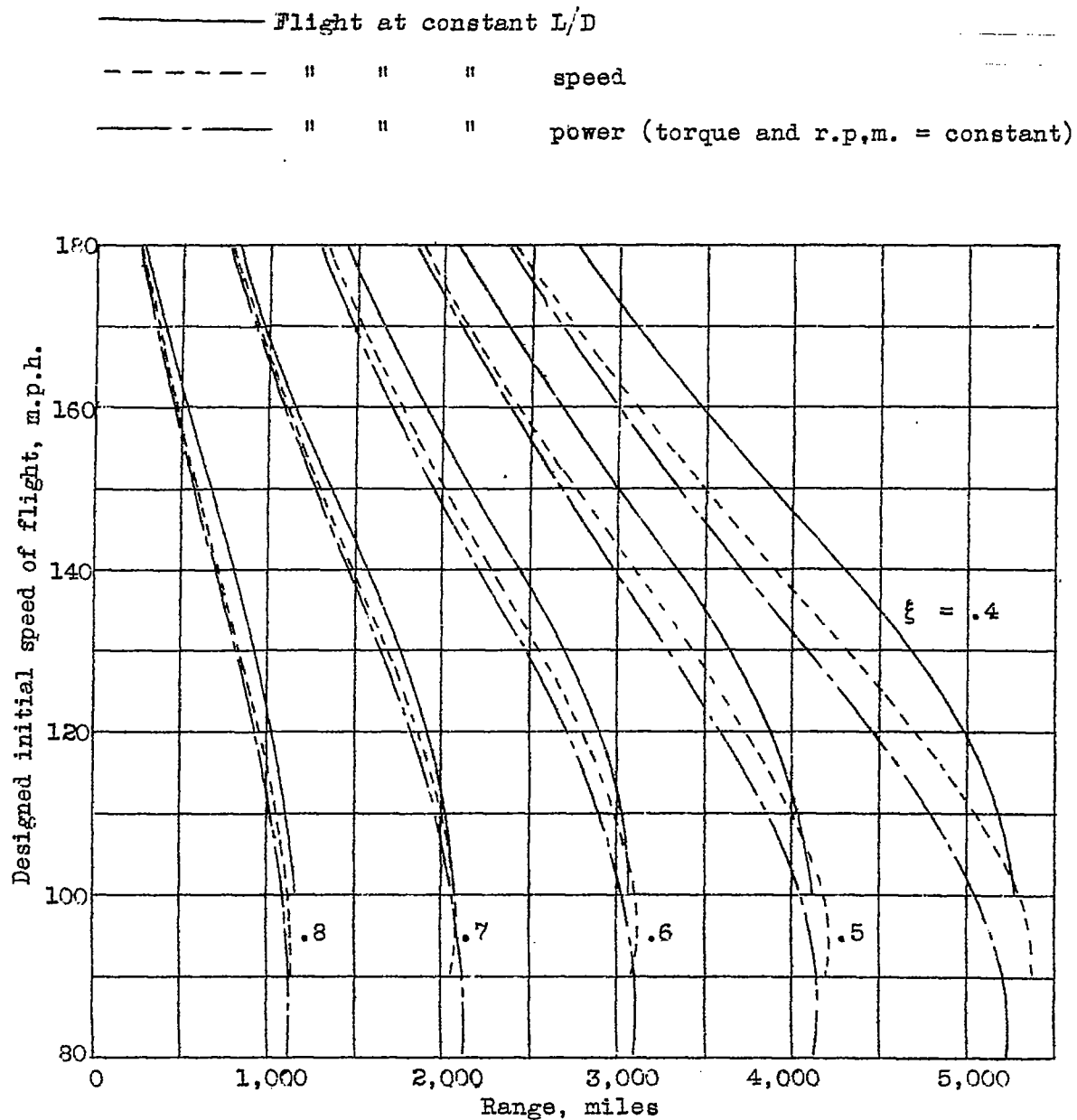


Figure 22.- Effect on range of different flight conditions.

——— Flight at constant L/D
 - - - " " " speed
 ——— " " " power

X Initial speed of flight = 100 m.p.h.

O " " " " 120 "
 □ " " " " 140 "
 Δ " " " " 160 "
 + " " " " 180 "

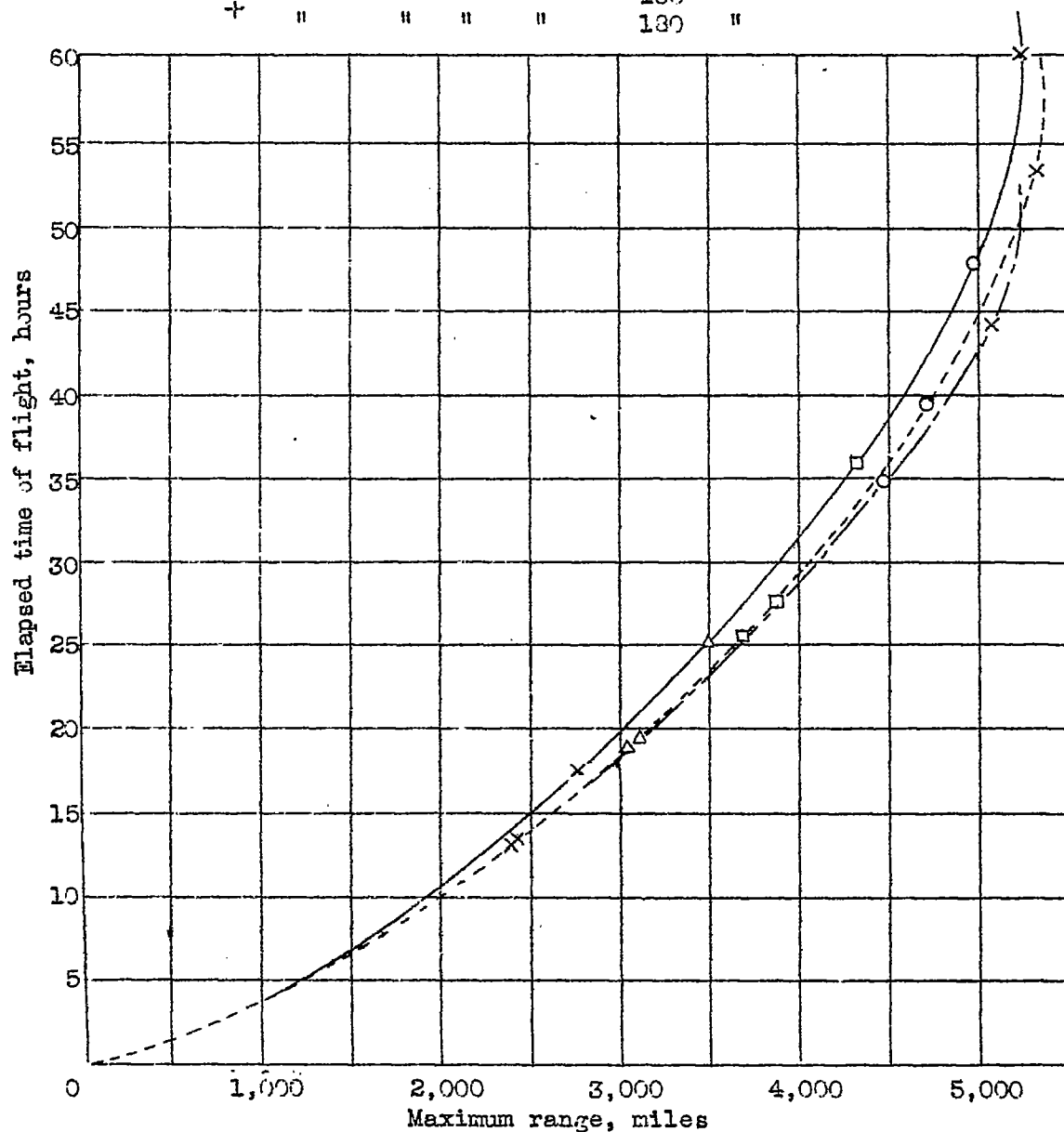


Figure 23.- Comparison of maximum range for given time for different methods of flight, $\xi = 0.4$.

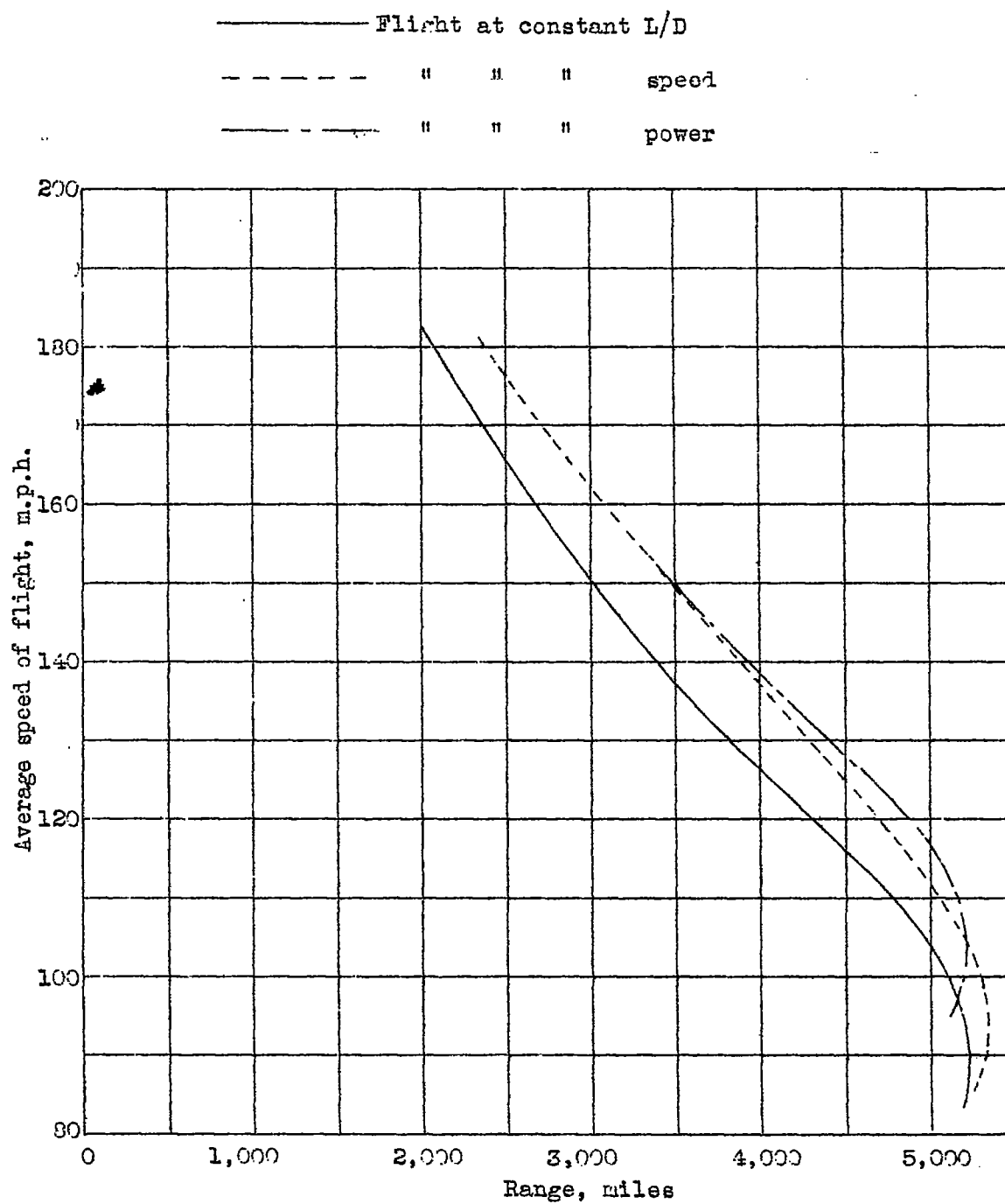


Figure 24.- Comparison of maximum range for different average speeds of flight for different methods of flight. $\xi = 0.4$.

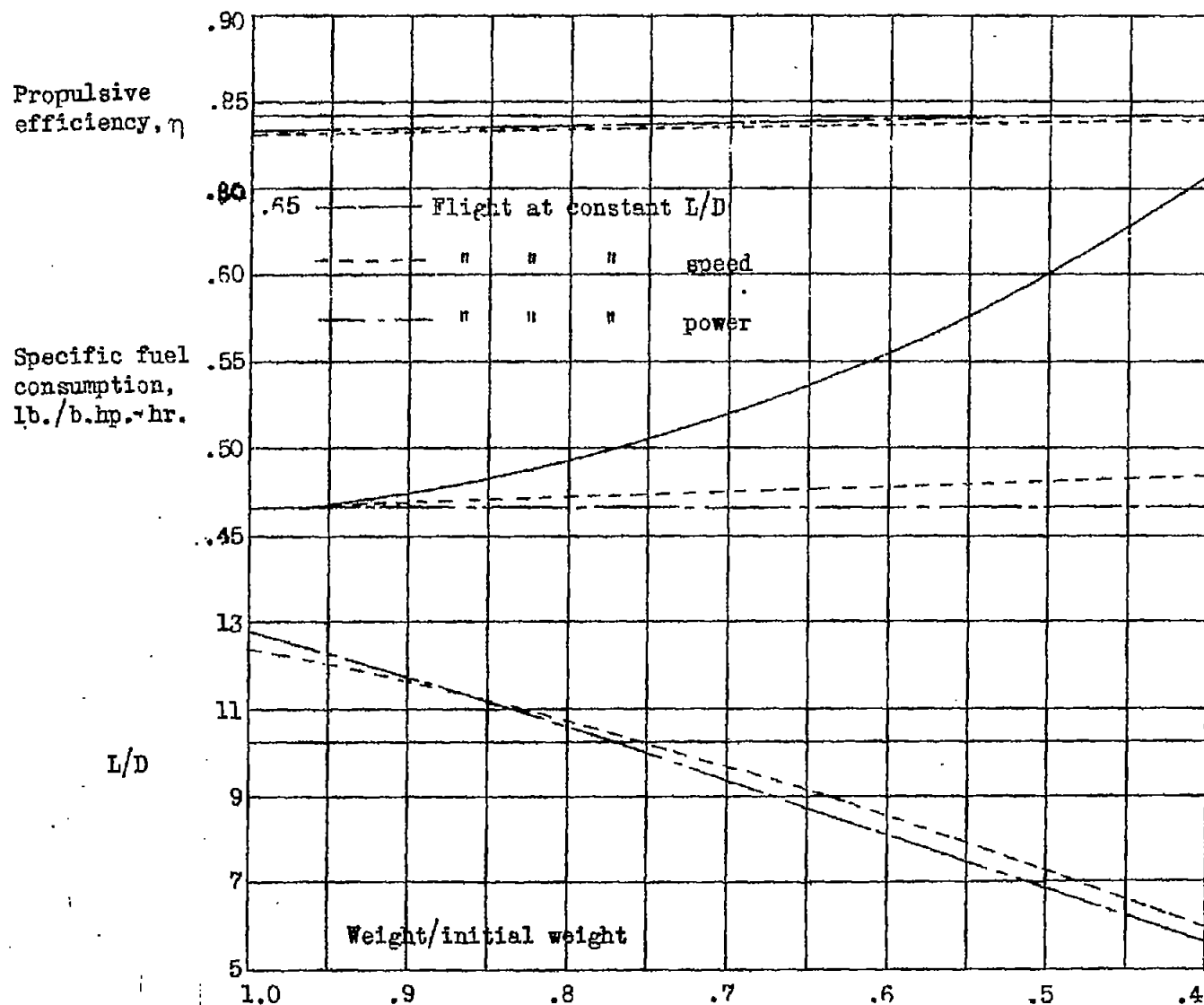


Figure 25.-
Changes in L/D,
specific fuel
consumption,
and propul-
sive efficien-
cy with de-
creasing load
as flight
progresses
for different
conditions
of flight.
Average speed
of flight =
140 m.p.h.

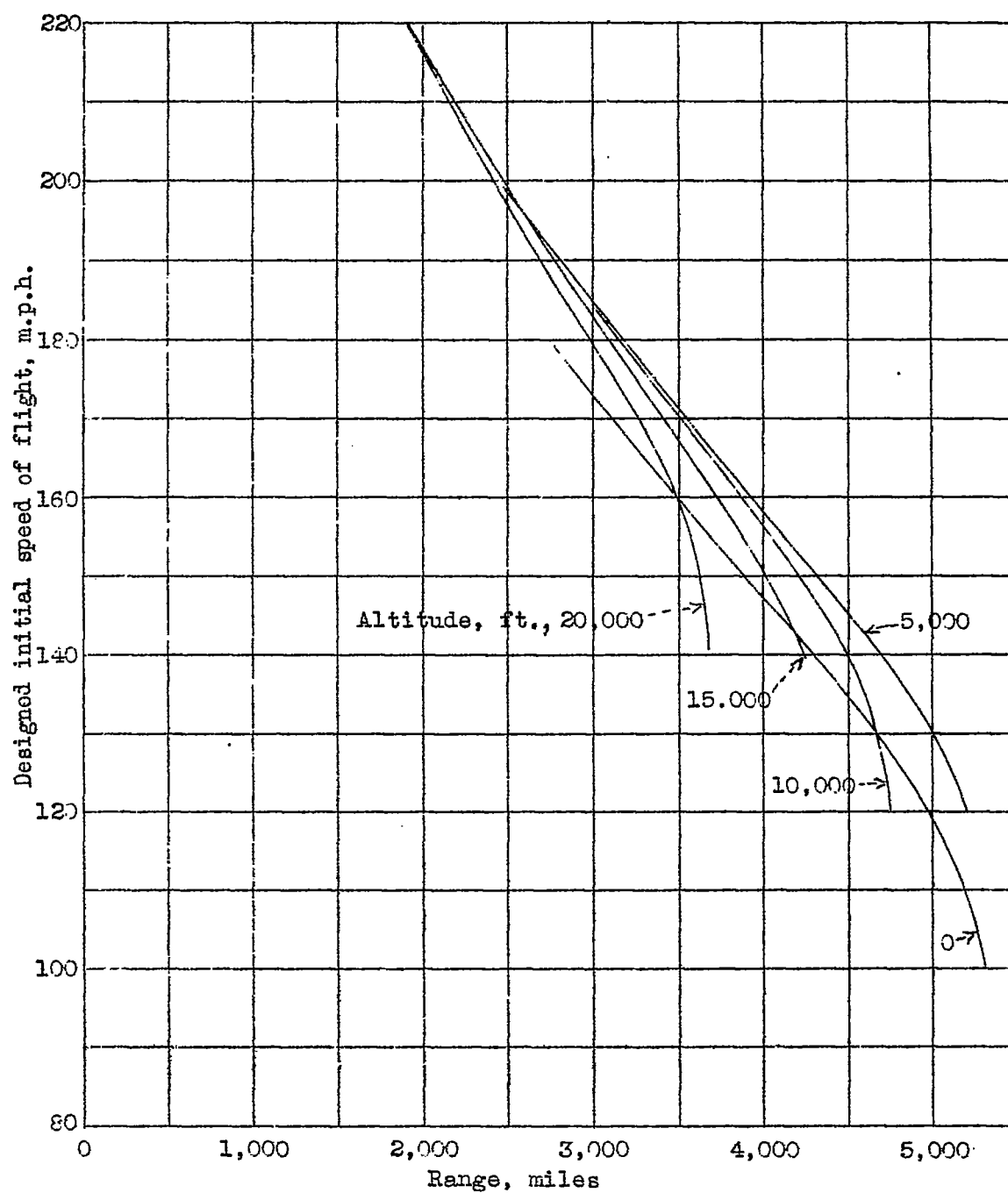


Figure 26.- Flight at various altitudes with unsupercharged engines. (Engines designed to develop 0.75 sea-level power at altitude of flight, $\xi = 0.4$.)

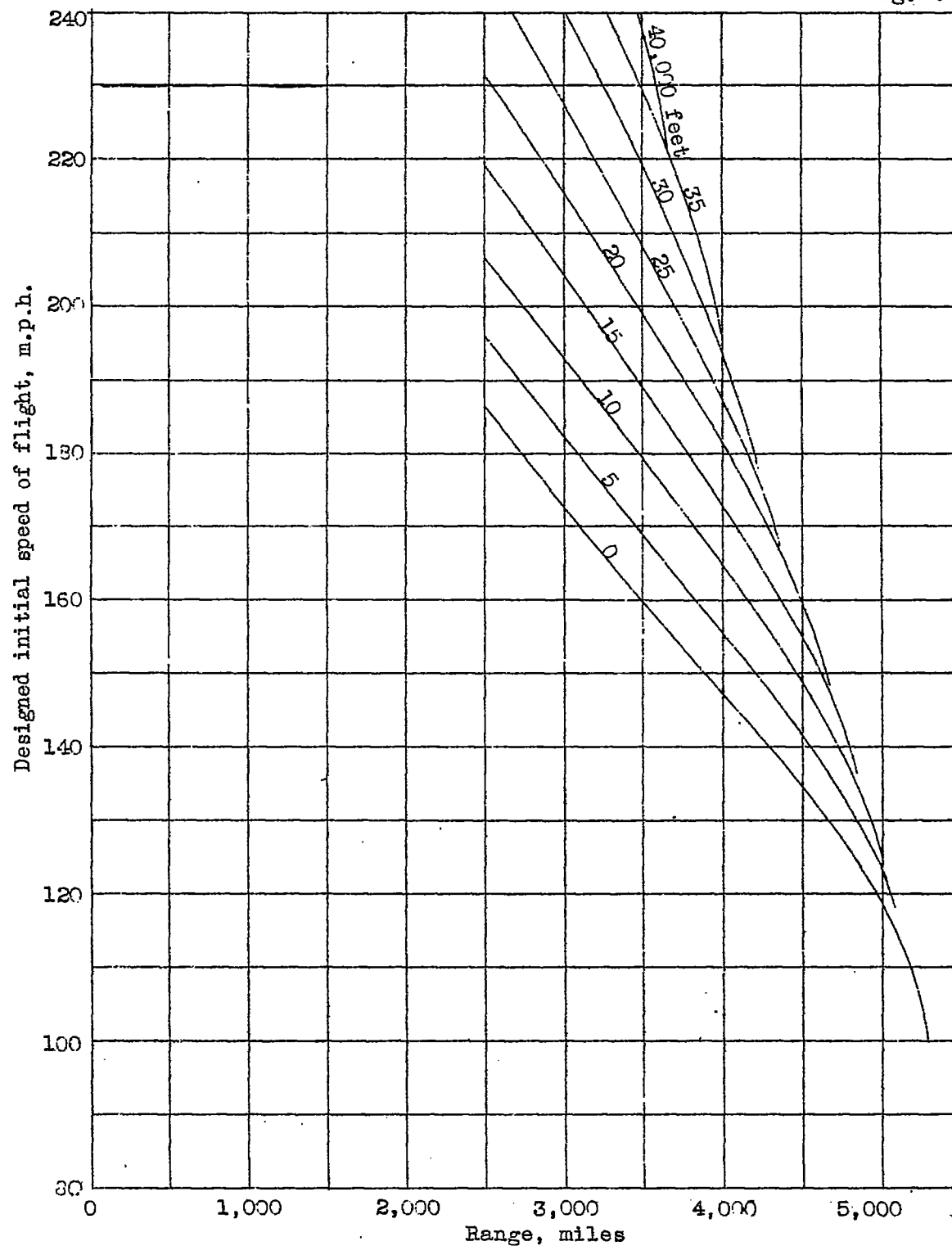


Figure 27.- Flight at various altitudes with engines supercharged to altitude of flight. (Engines equipped with geared centrifugal superchargers, $\xi = 0.4$.)

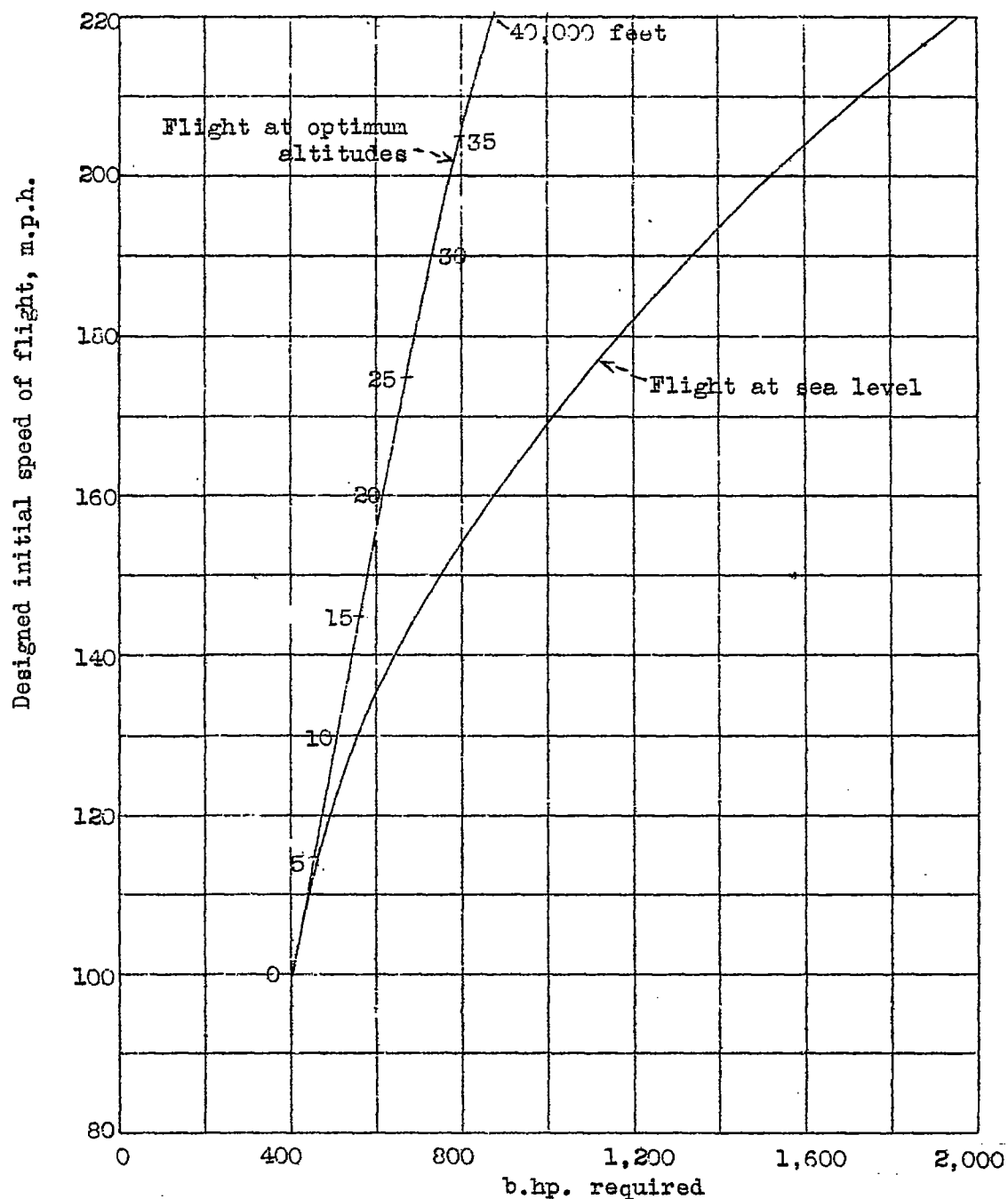
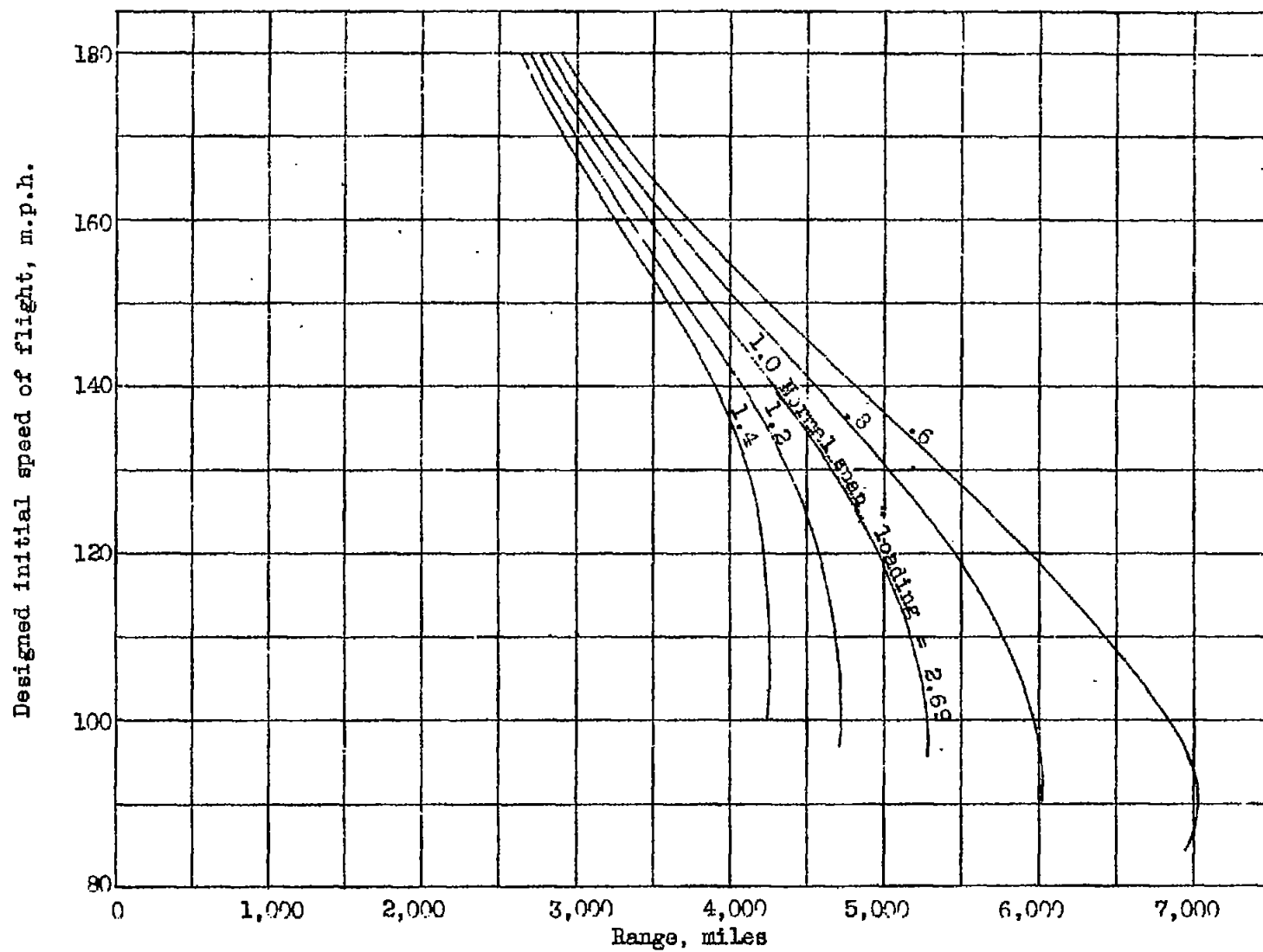


Figure 28.—Power required to fly at various speeds at sea level
and at the optimum altitudes with supercharged engines.

Figure 29.- Effect of span loading on range. $\xi = 0.4$.

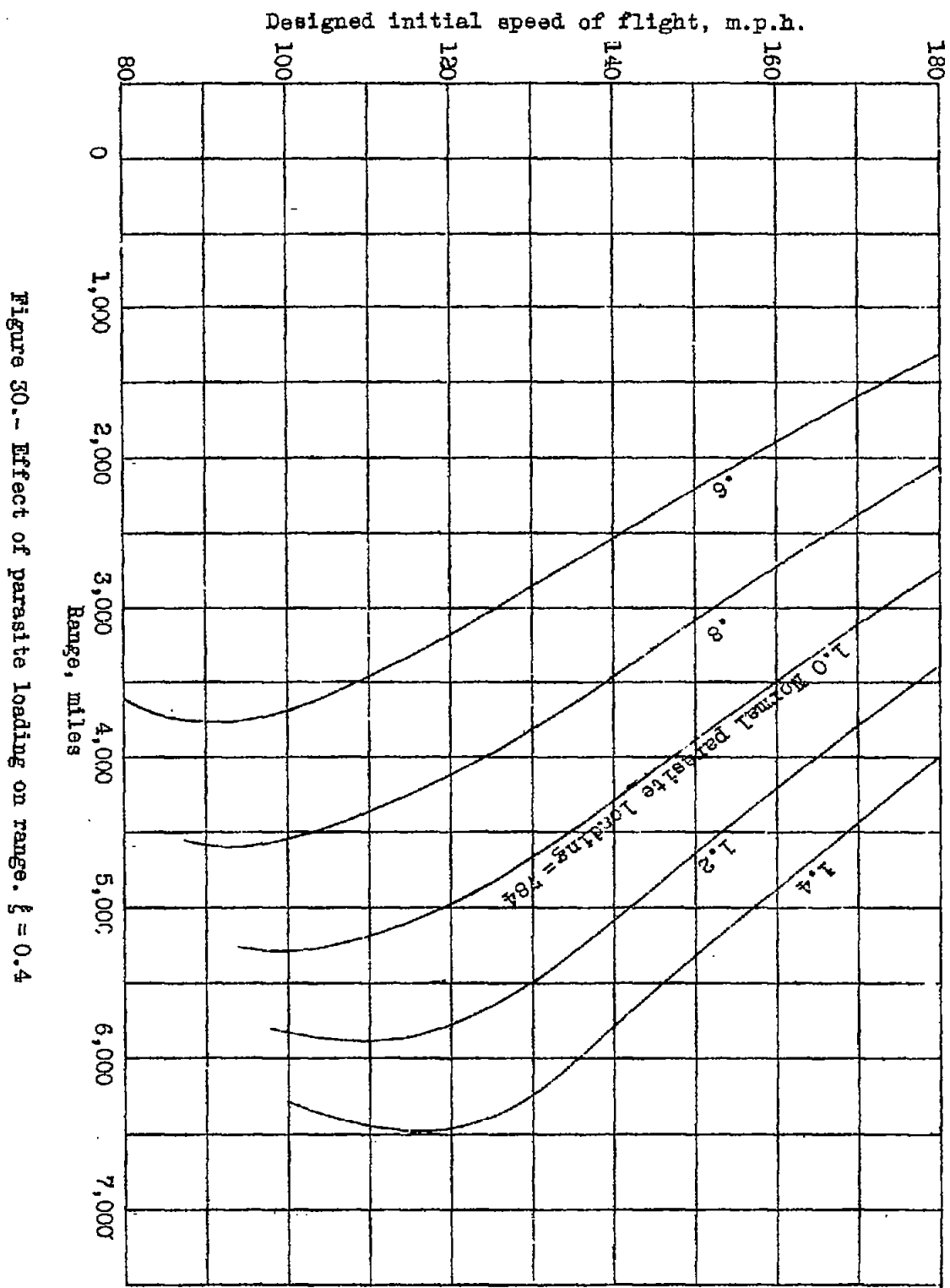


Figure 30.- Effect of parasite loading on range. $\xi = 0.4$

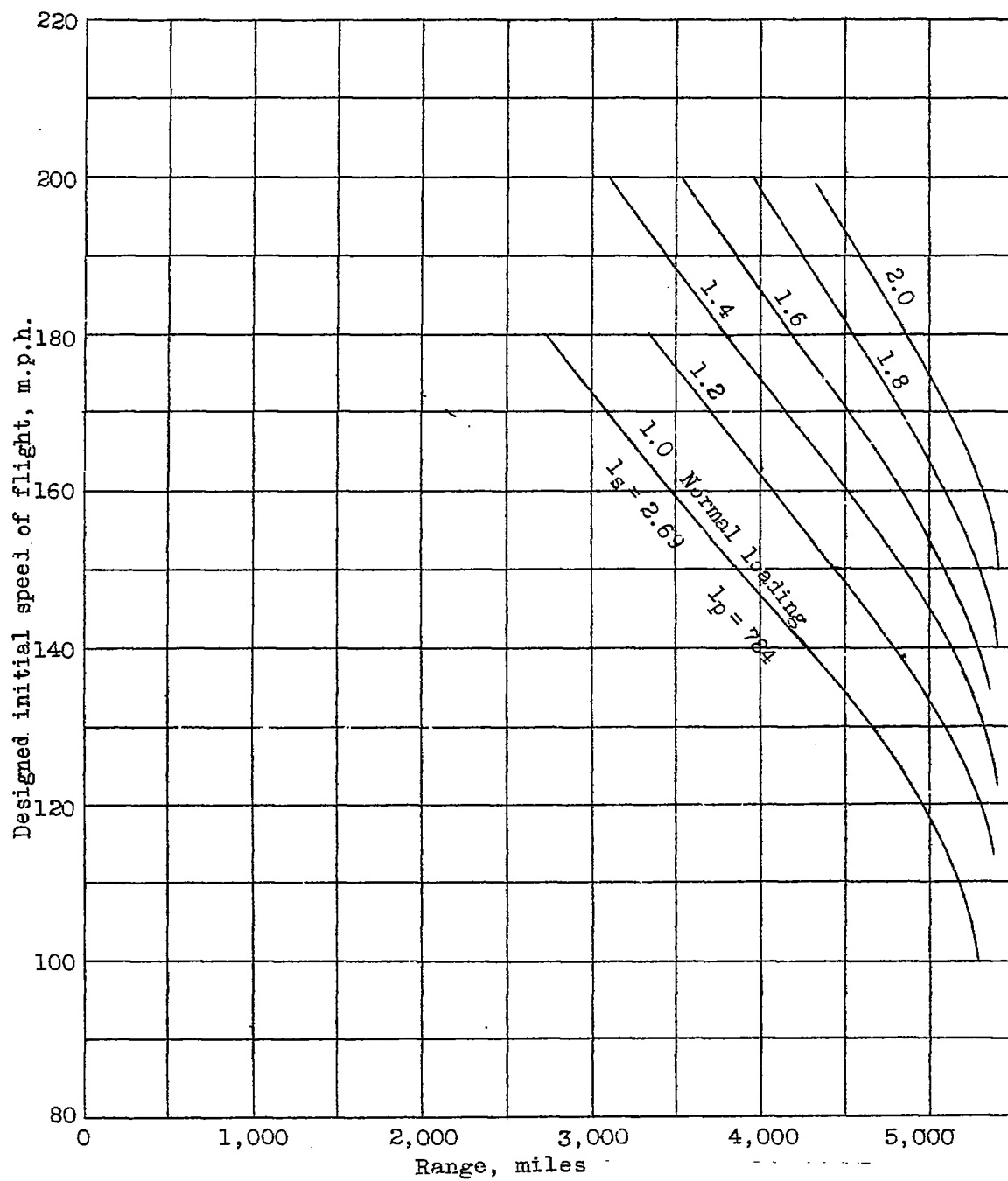


Figure 31.- Effect of both span loading and parasite loading on range. $\xi = 0.4$

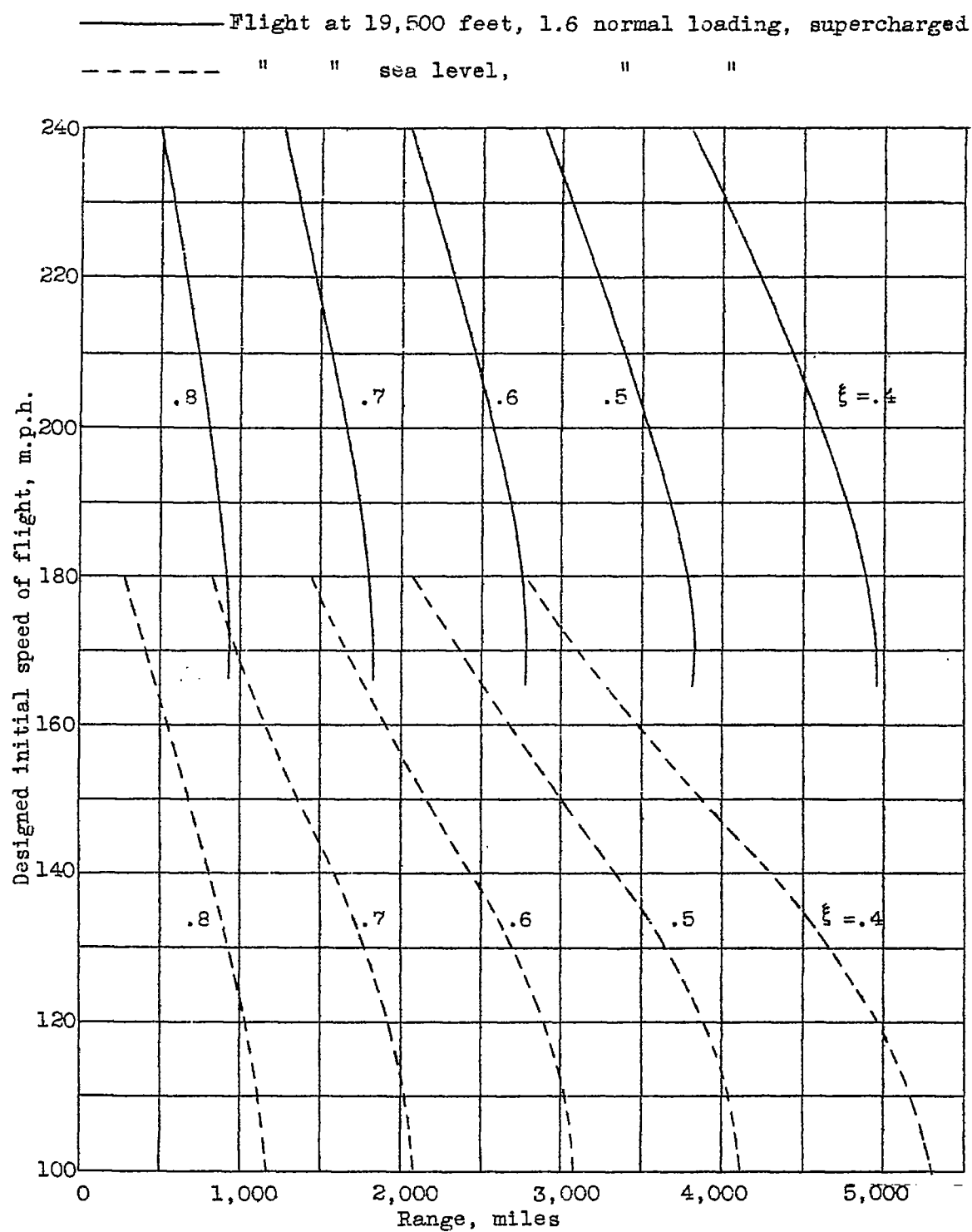


Figure 32.- Flight at 19,500 feet, supercharged to 15,000 feet, span and parasite loadings 1.6 normal.

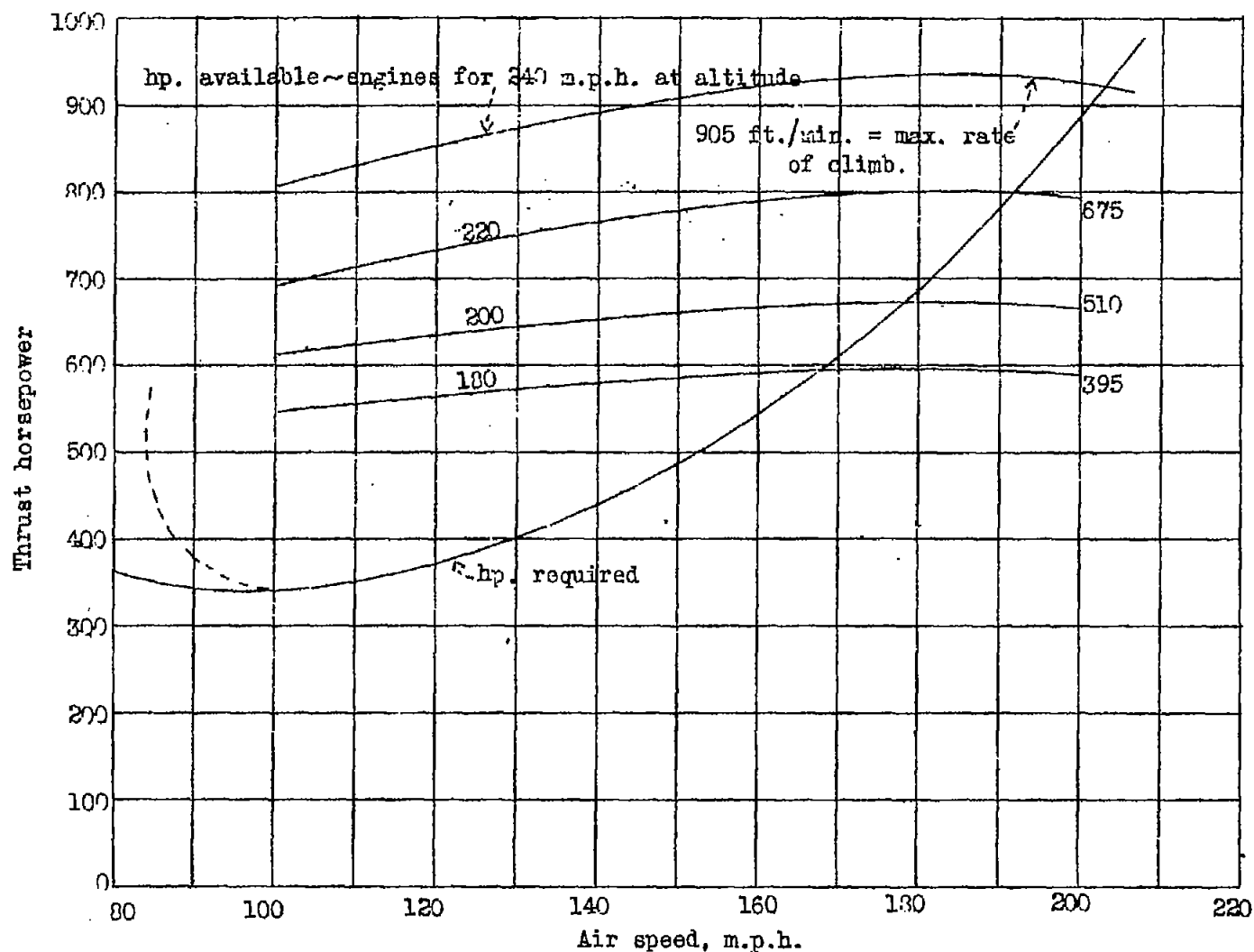
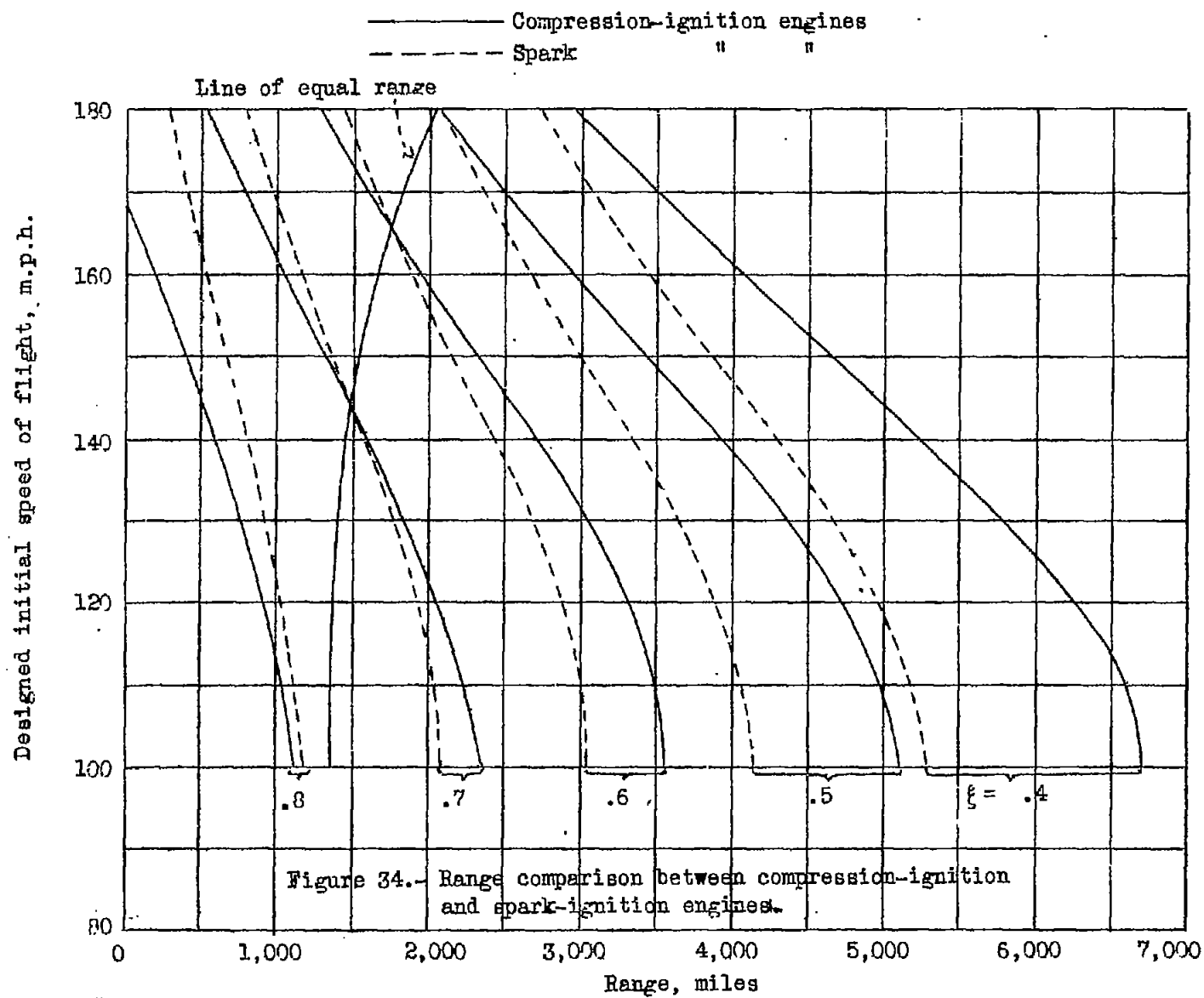
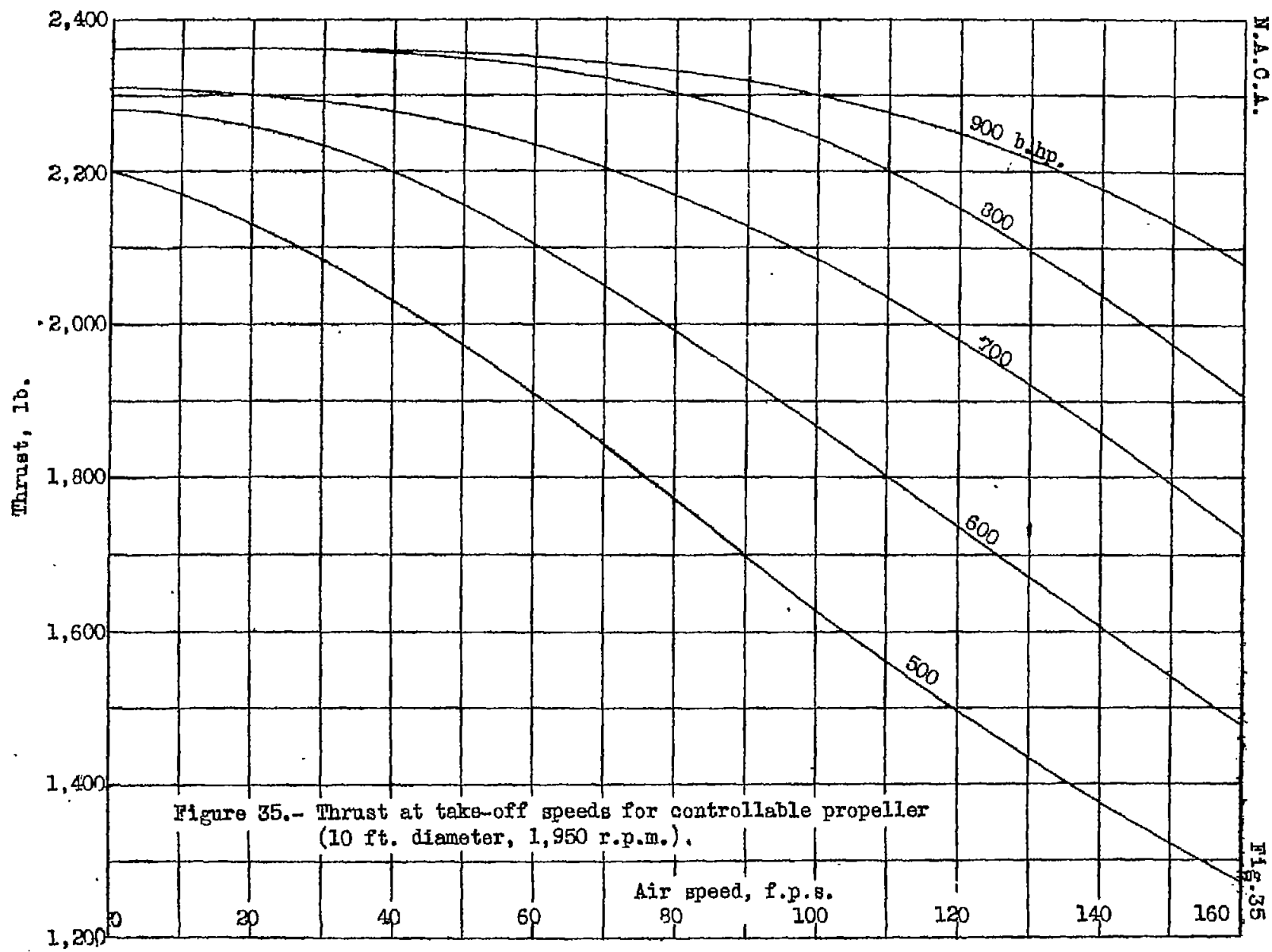


Figure 33.-- Sea-level power-available and power-required curves for airplane designed to fly at 19,000 ft. and with 1.6 normal loading. (Controllable propellers used.)





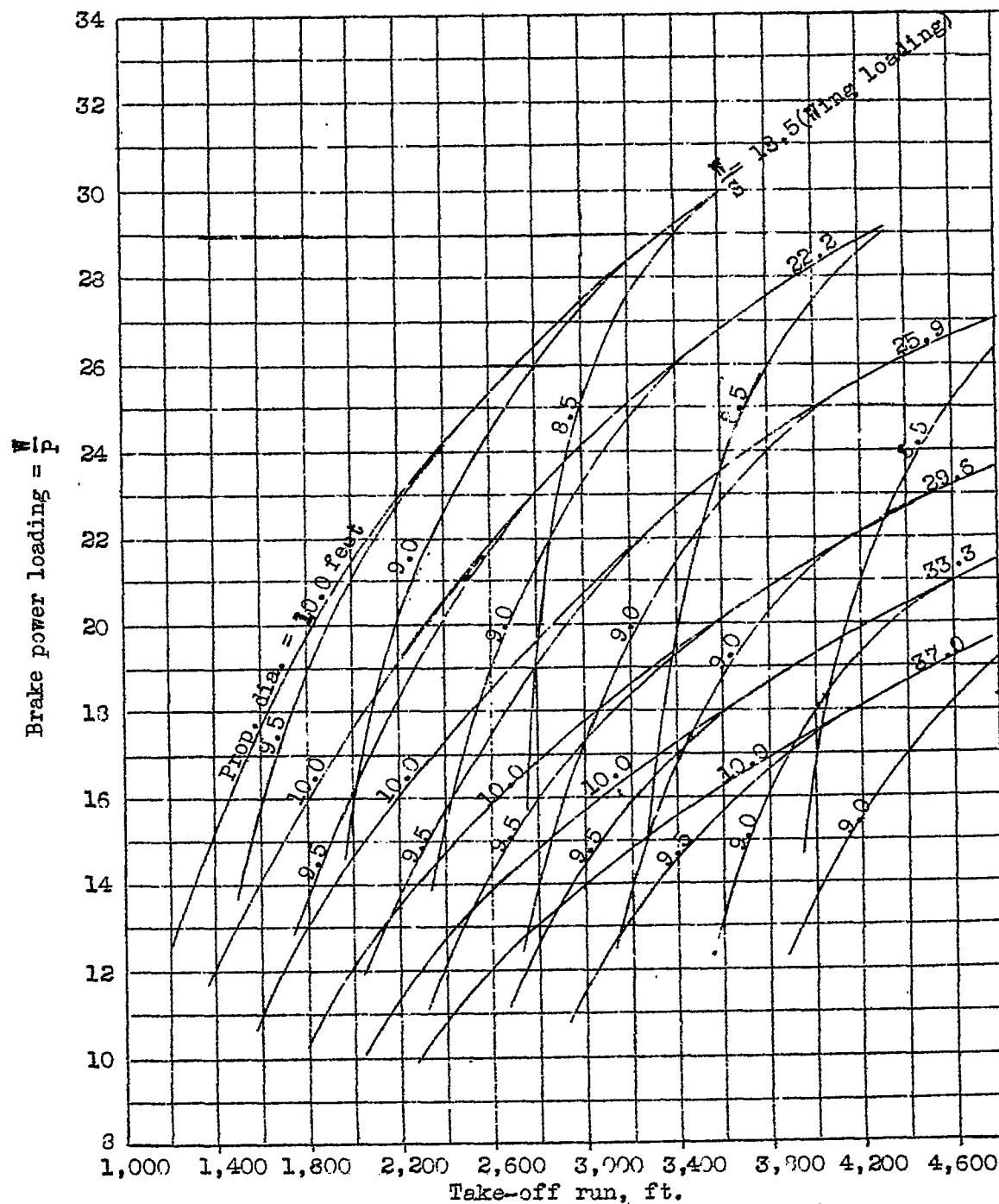


Figure 36.- Relationship between take-off run, power loading, wing loading, and propeller diameter for a two-engine airplane of 17,500 lb. weight. (Two controllable propellers used, engine speed, 1,950 r.p.m. W/S of 18.5 corresponds to normal loading.)

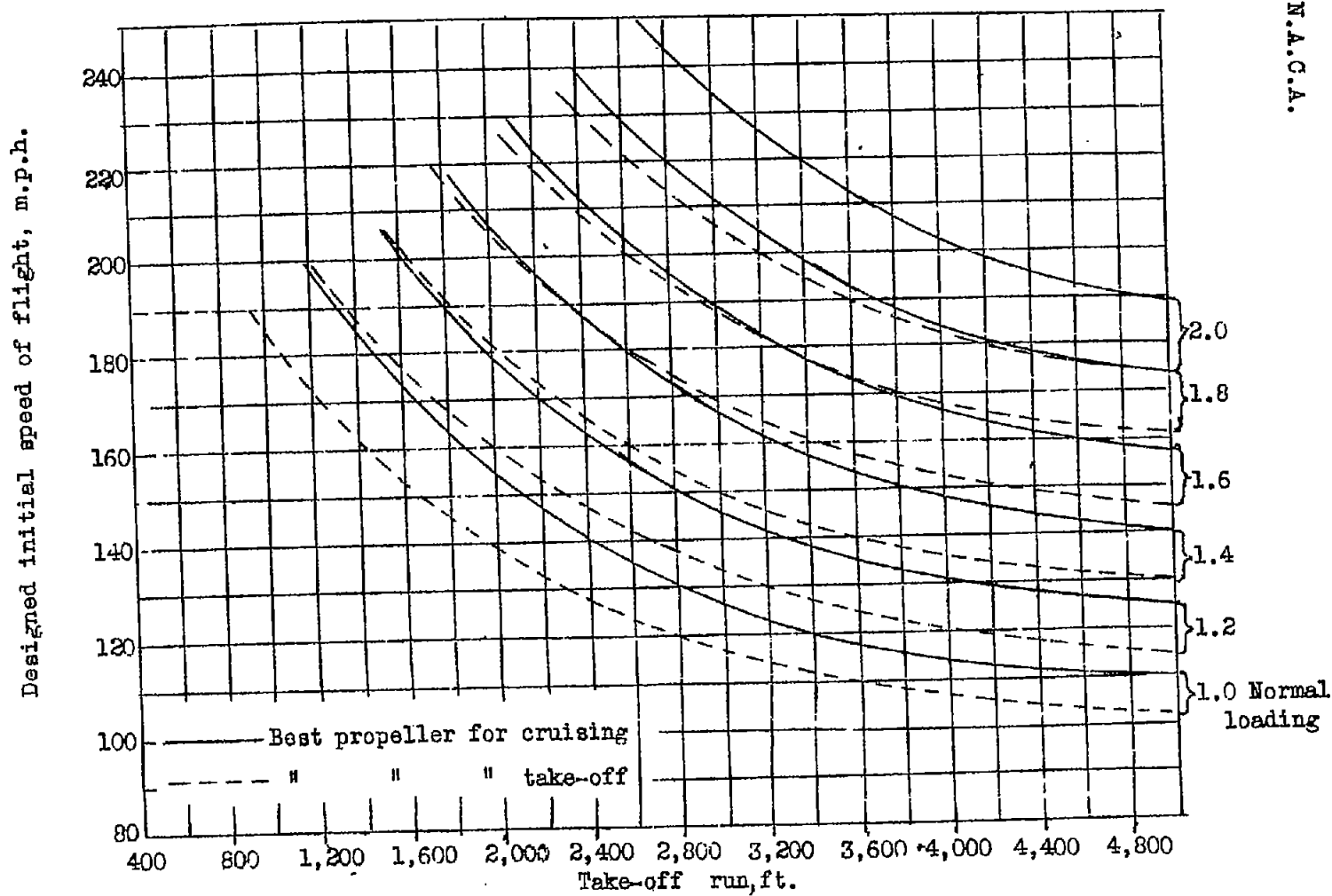


Figure 37.- Take-off runs for different designed loadings at flight speeds.

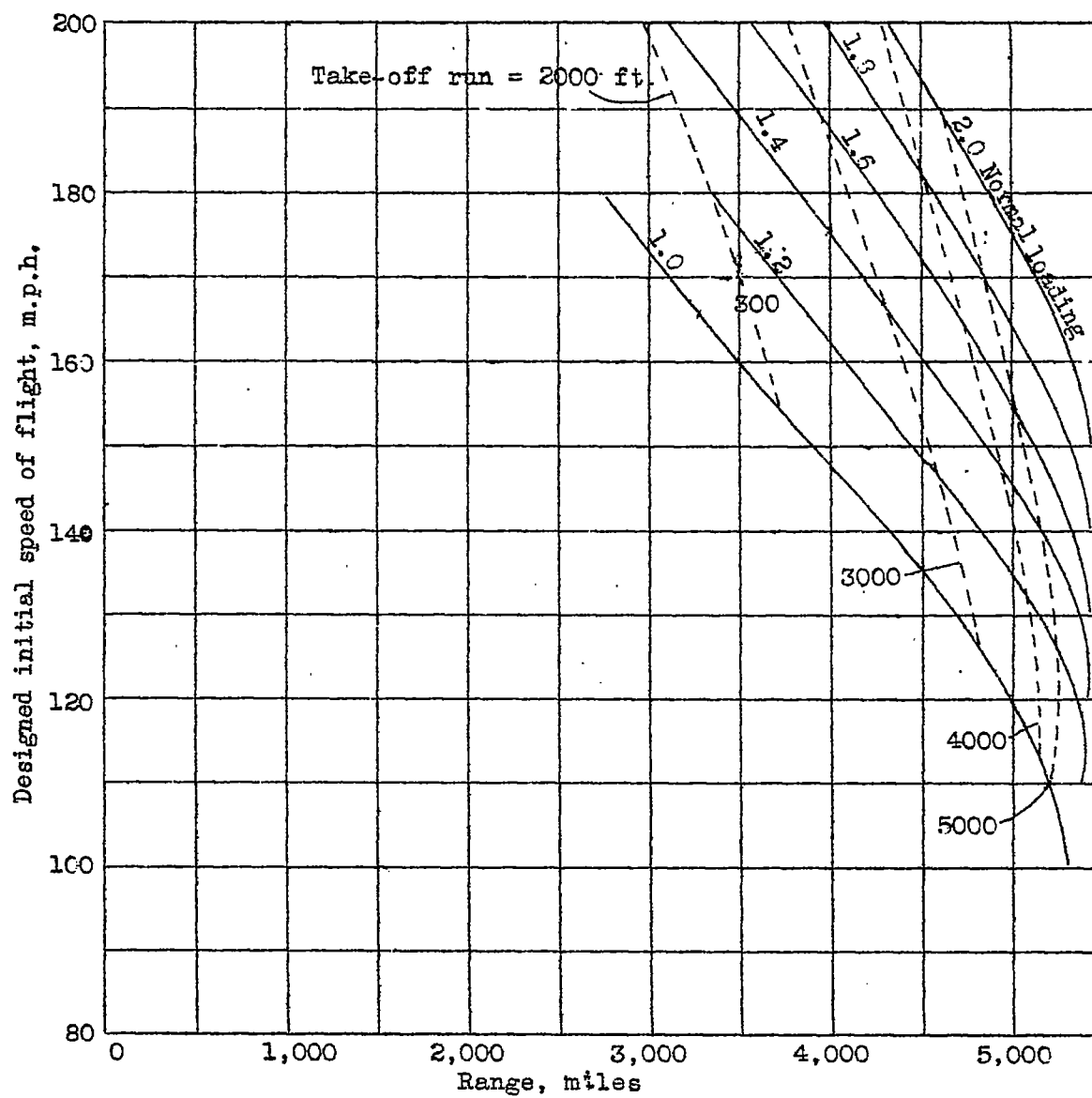


Figure 38.--Length of take-off run corresponding to different designed loading conditions. $\xi = 0.4$

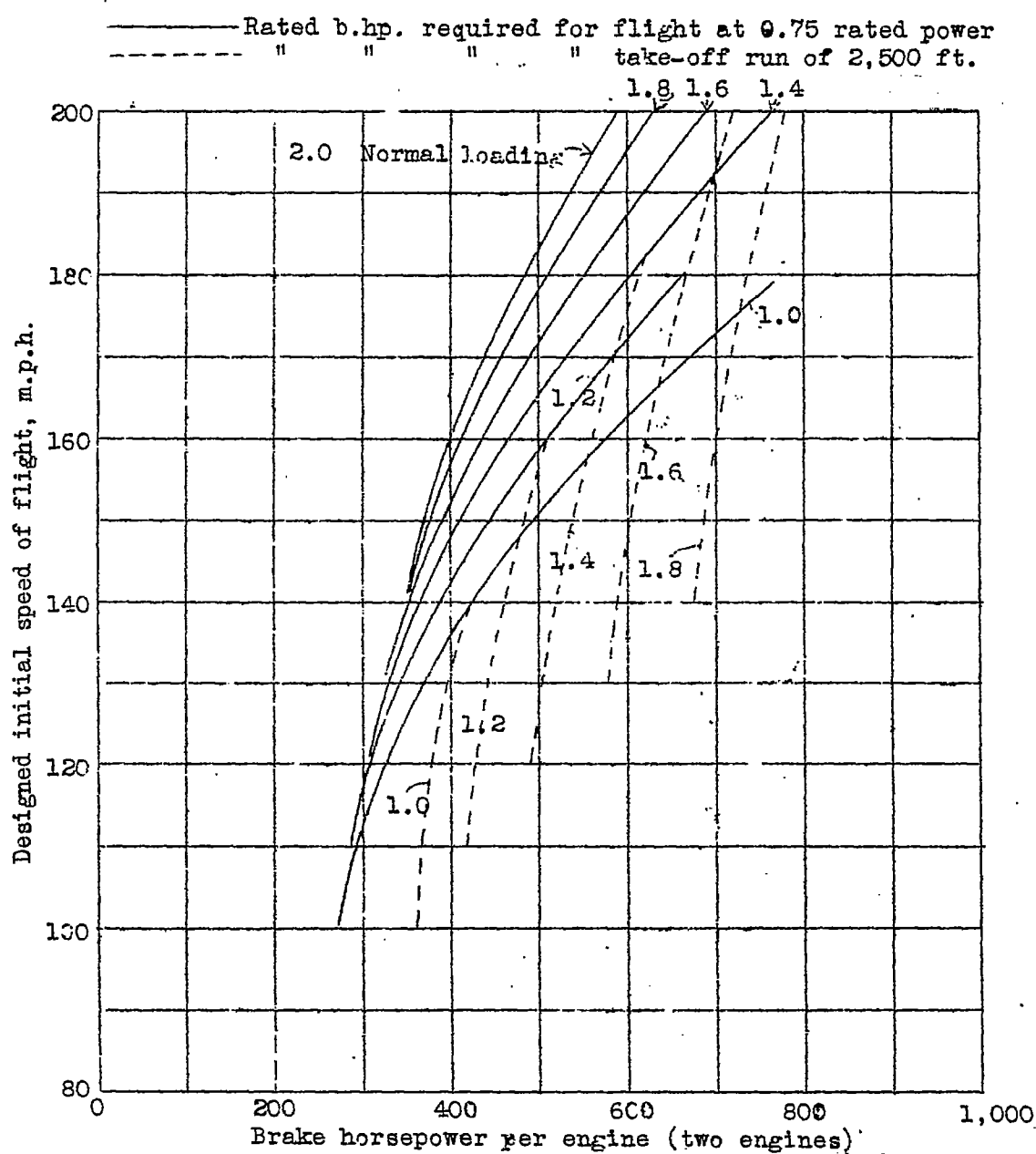


Figure 39.—Brake horsepower required for flight and brake horsepower required for take-off run of 2,500 ft.

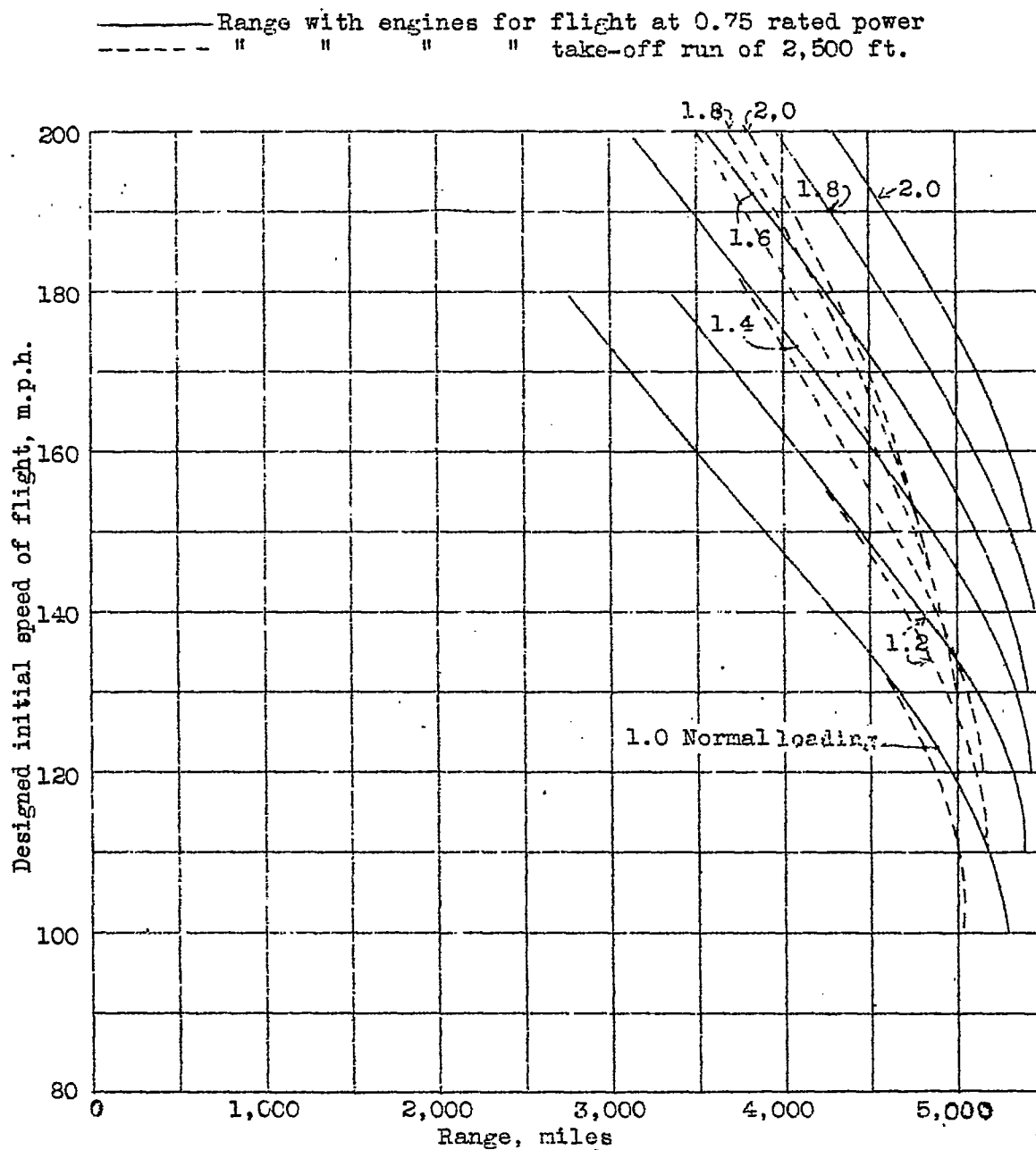


Figure 40.—Effect of increasing the engine size in order to make take-offs within 2,500 ft. $\xi = 0.4$

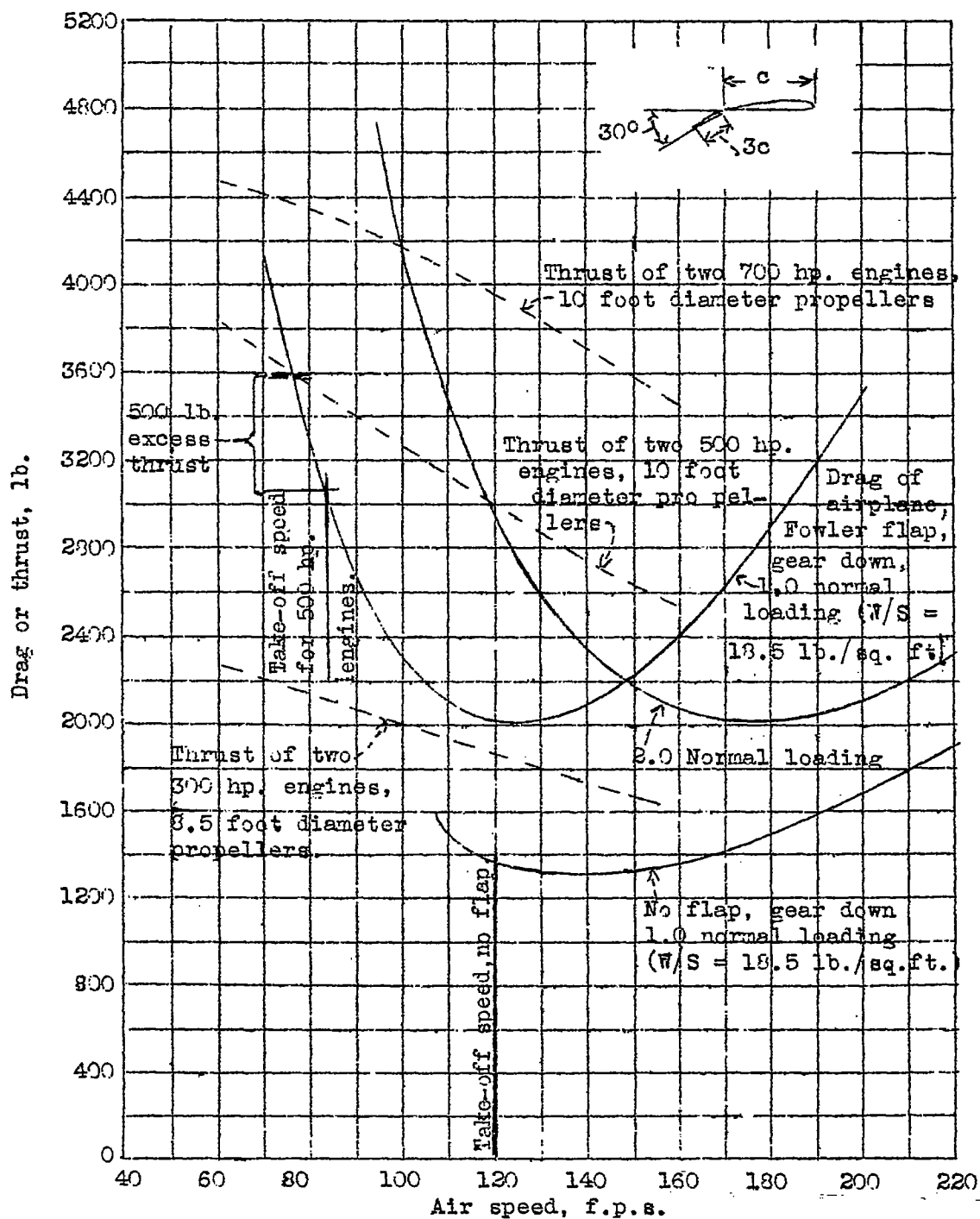


Figure 41.- Comparison of drag of airplane with and without flaps, illustrating the differences in take-off speeds for the two conditions.

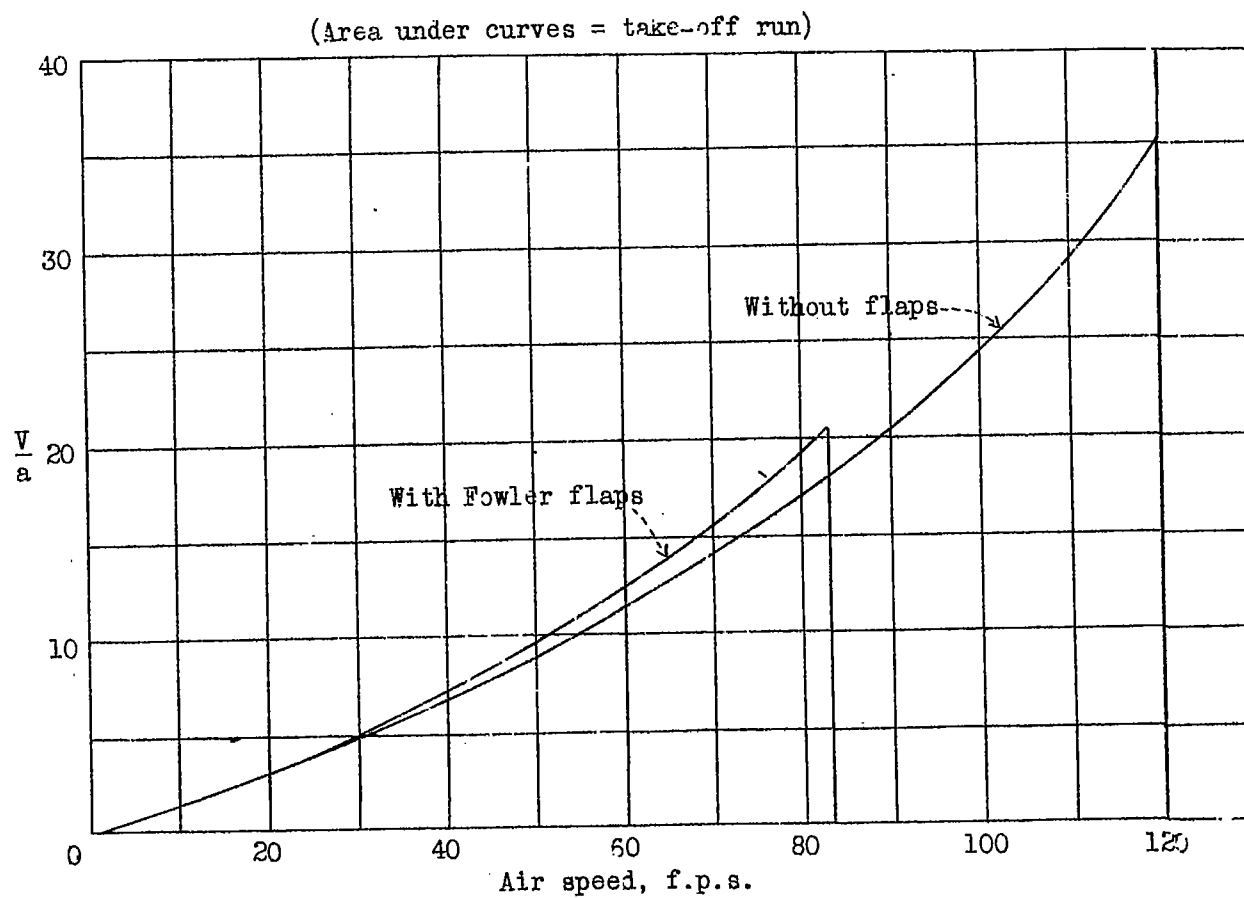


Figure 42.- Diagrams illustrating the effect of flaps on take-off run. (Two 500 hp. engines, 10 ft. diameter propellers.)

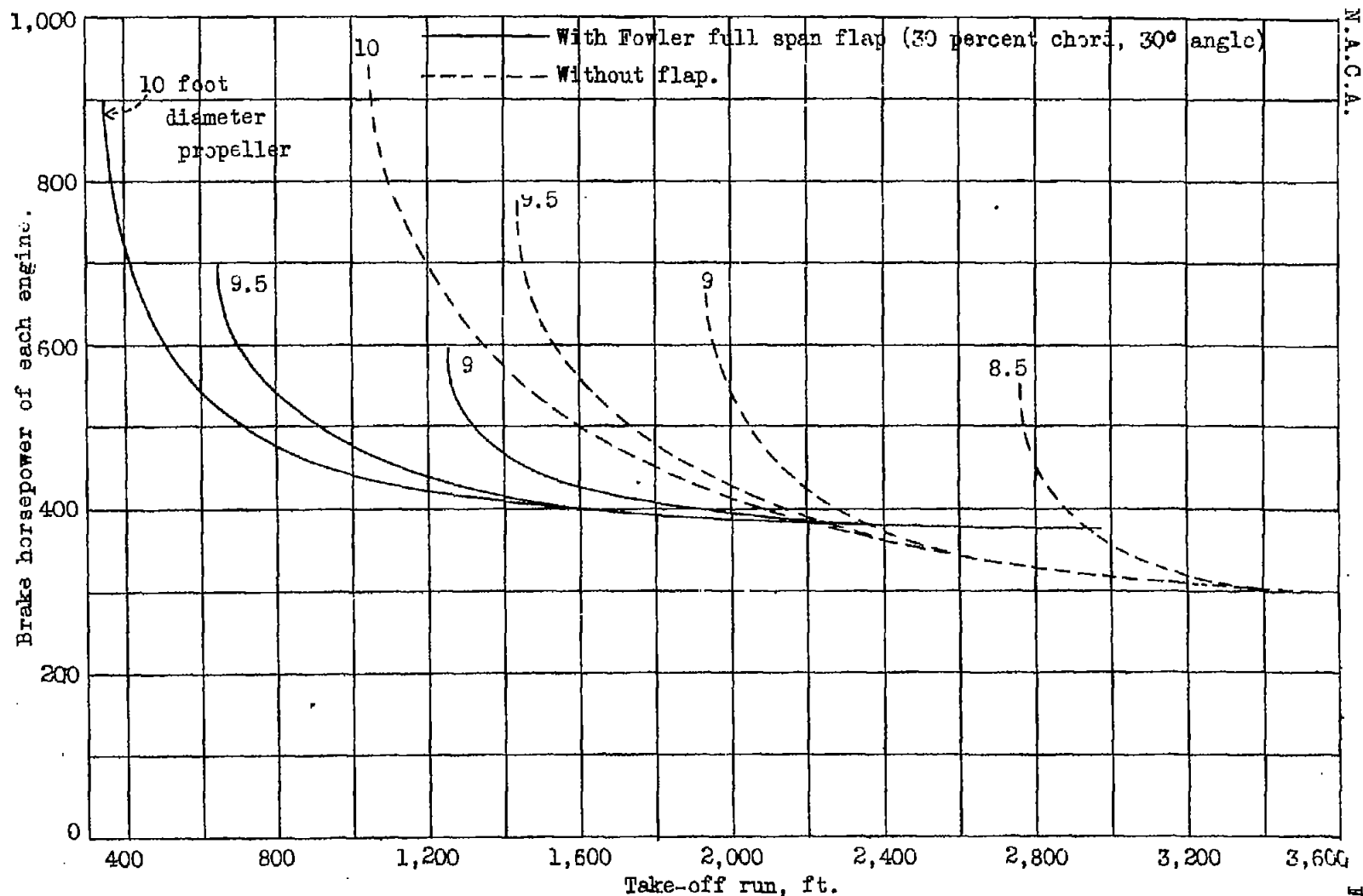


Figure 43.- Comparison of take-off runs with and without flaps. (1.0 normal loading, $W/S = 18.5$ lb./sq.ft. Two engines, 17,500 lb. gross weight.)

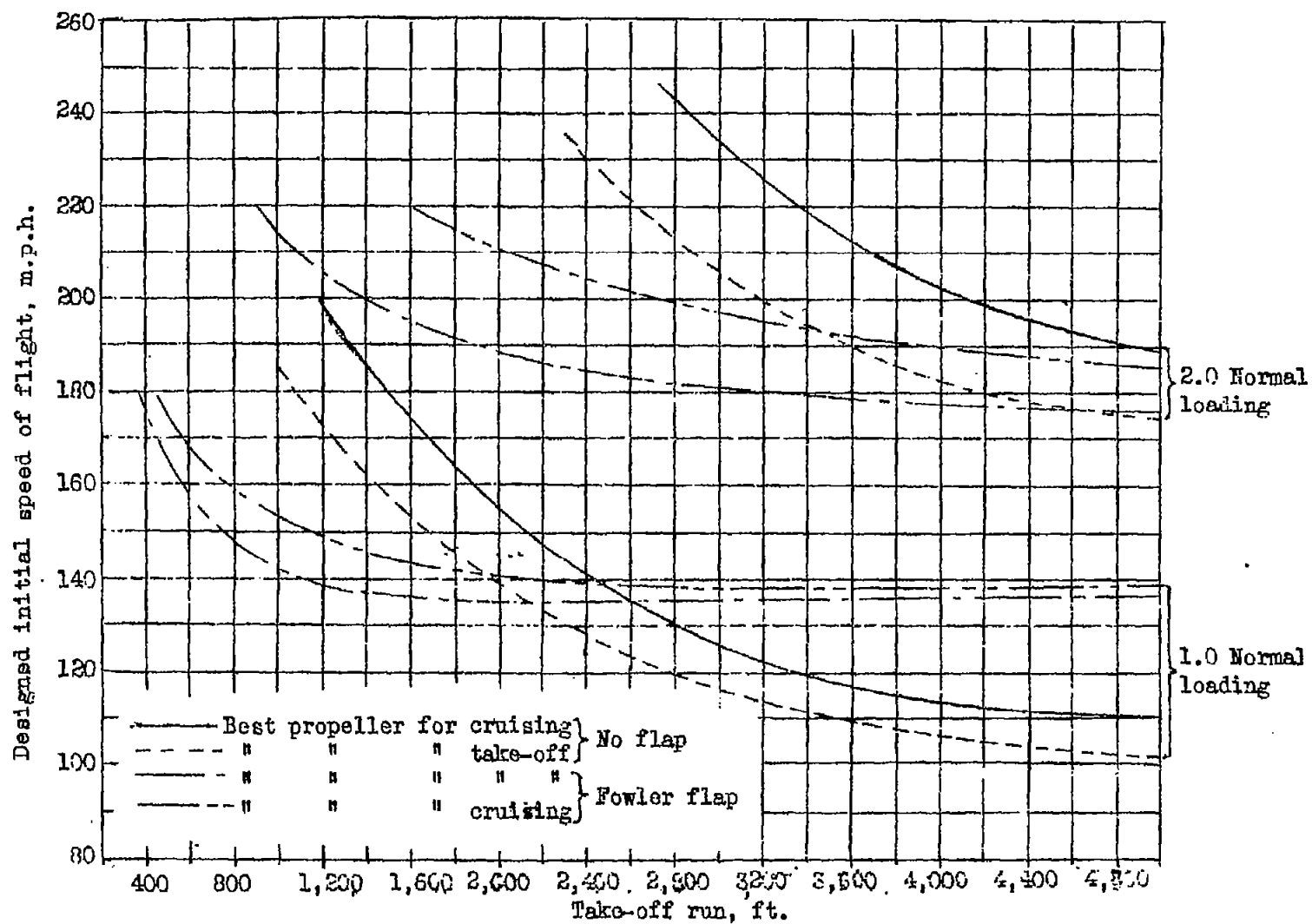


Figure 44.- Comparison of take-off runs with and without flaps for differently designed conditions.

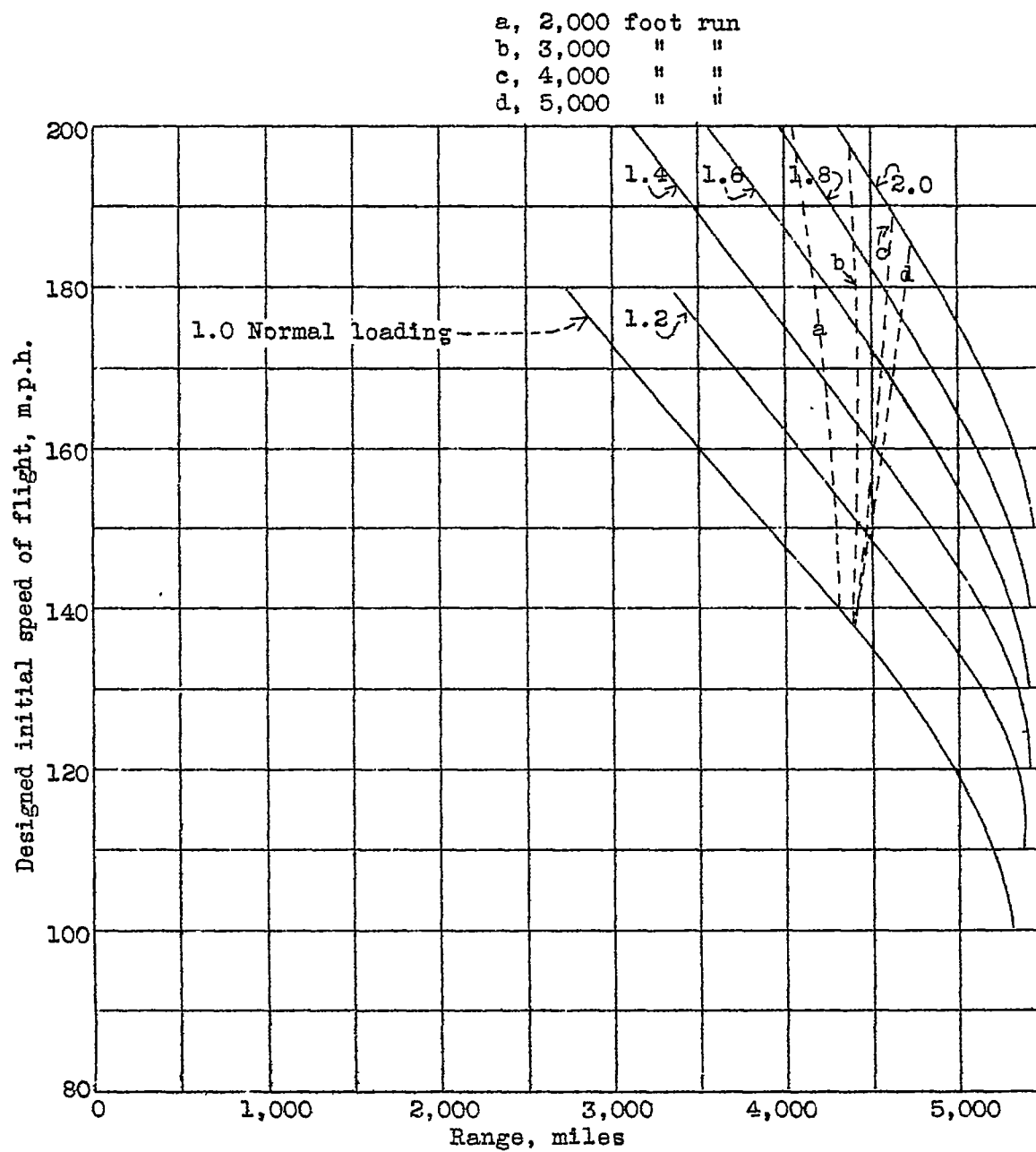


Figure 45.-Length of take-off run with flaps corresponding to differently designed loading conditions. $\xi = 0.4$

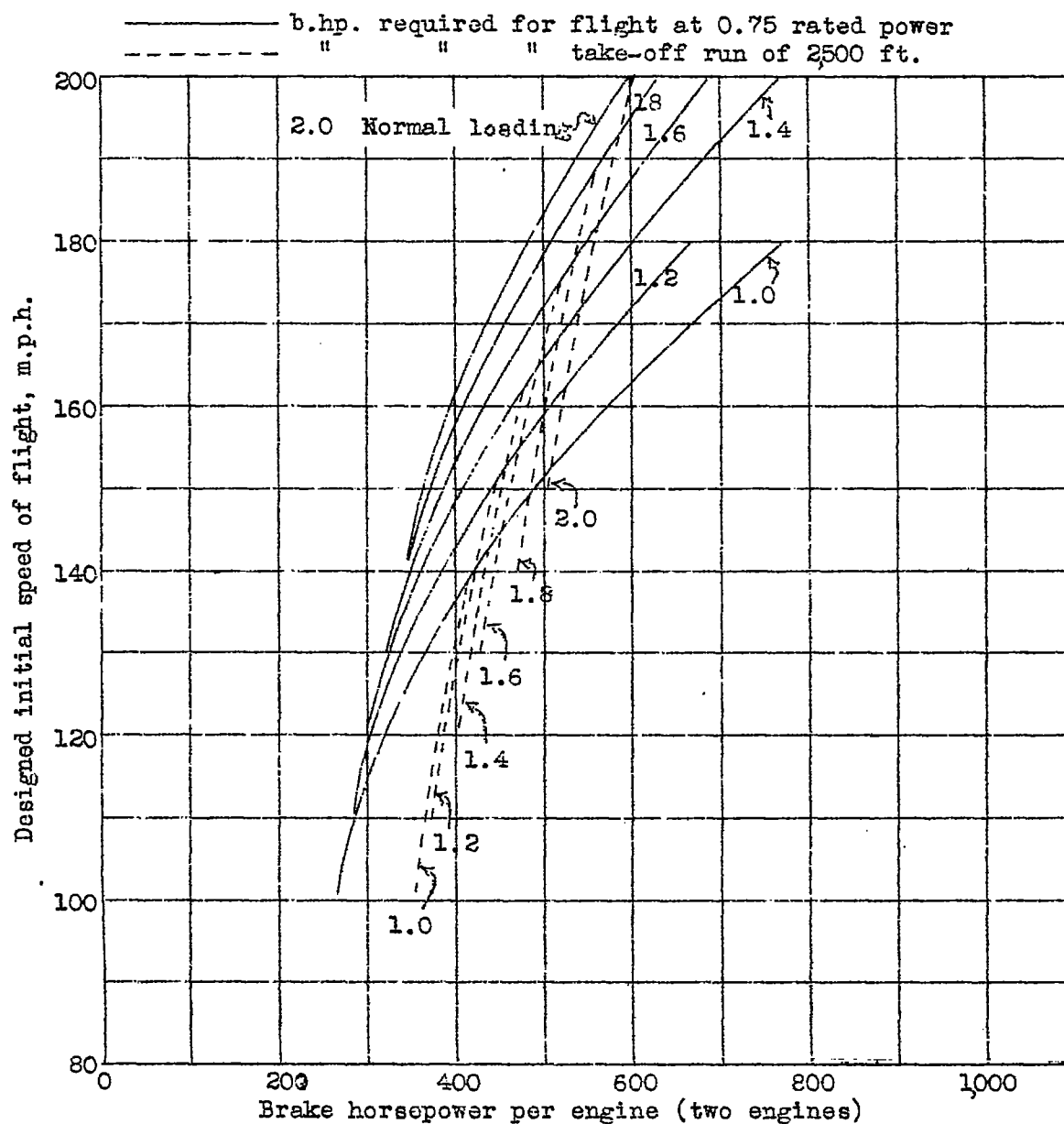


Figure 46.—Brake horsepower required for flight and brake horsepower necessary for take-off run of 2,500 ft. with flaps.

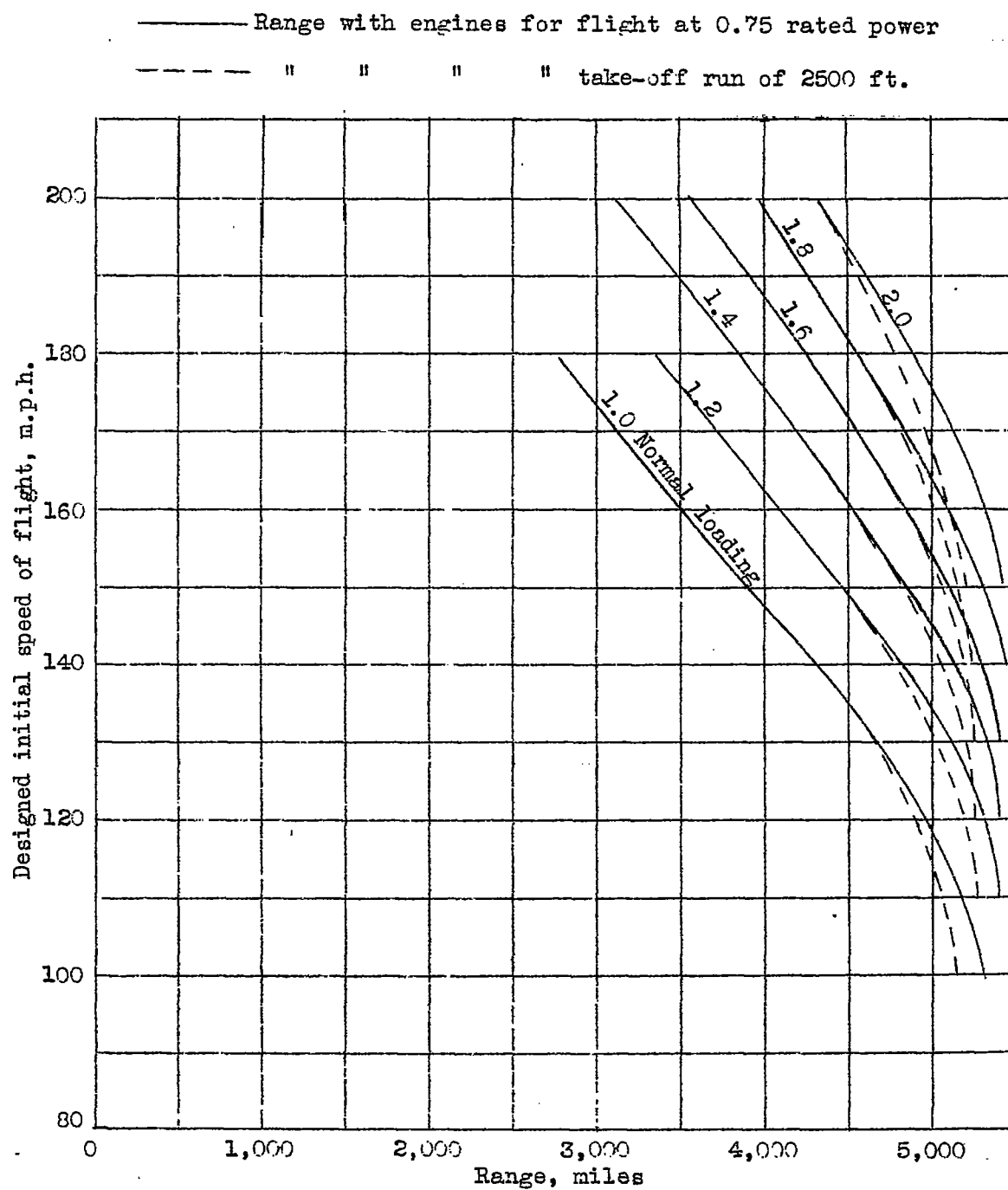


Figure 47.- Effect of increasing the engine size in order to make take-offs within 2,500 feet with flaps. $\xi = 0.4$.

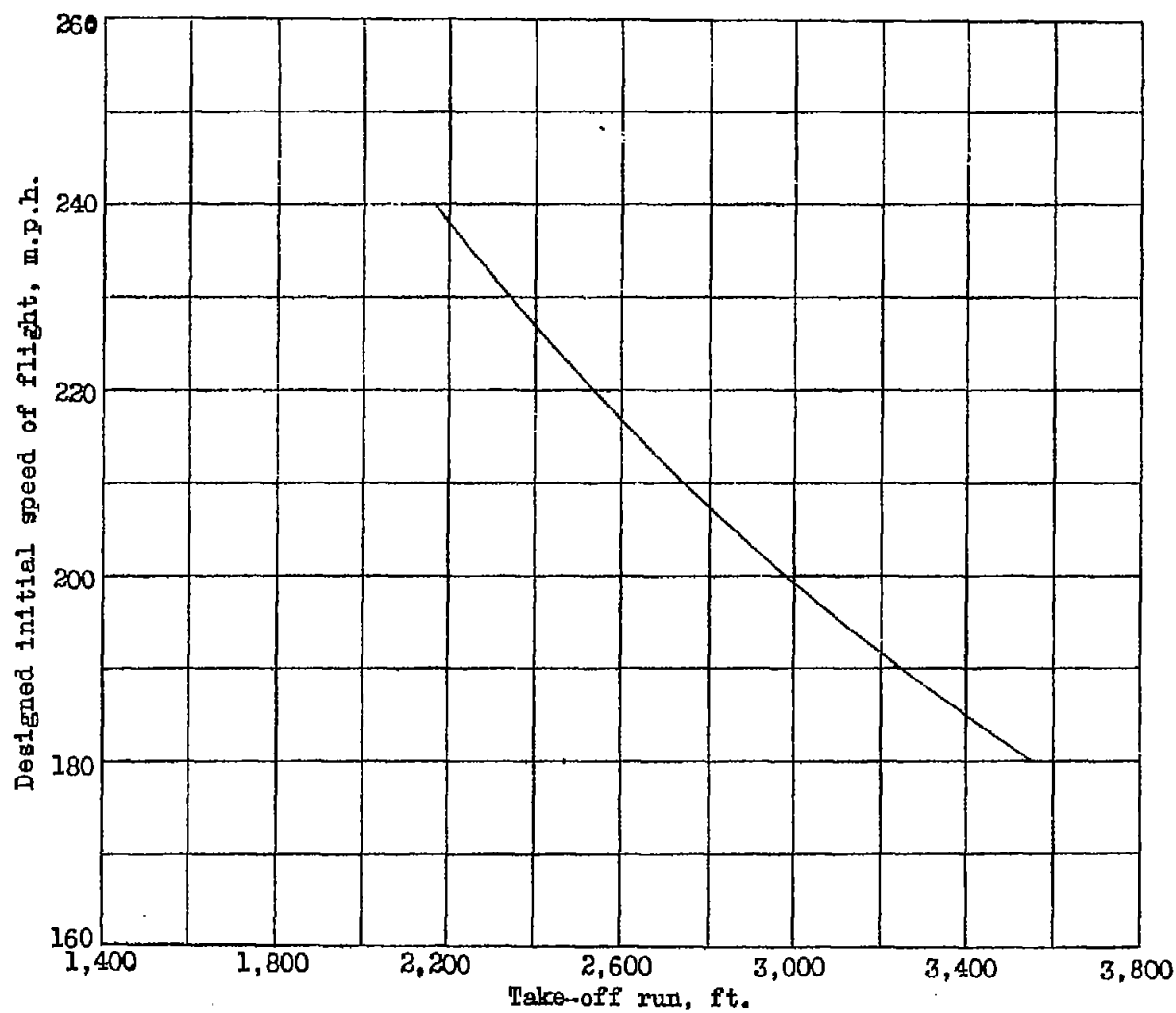


Figure 48.—Take-off run for airplane designed for composite condition. 1.6 normal loading; supercharged to an altitude of 15,000 ft; flight at 19,500 ft.

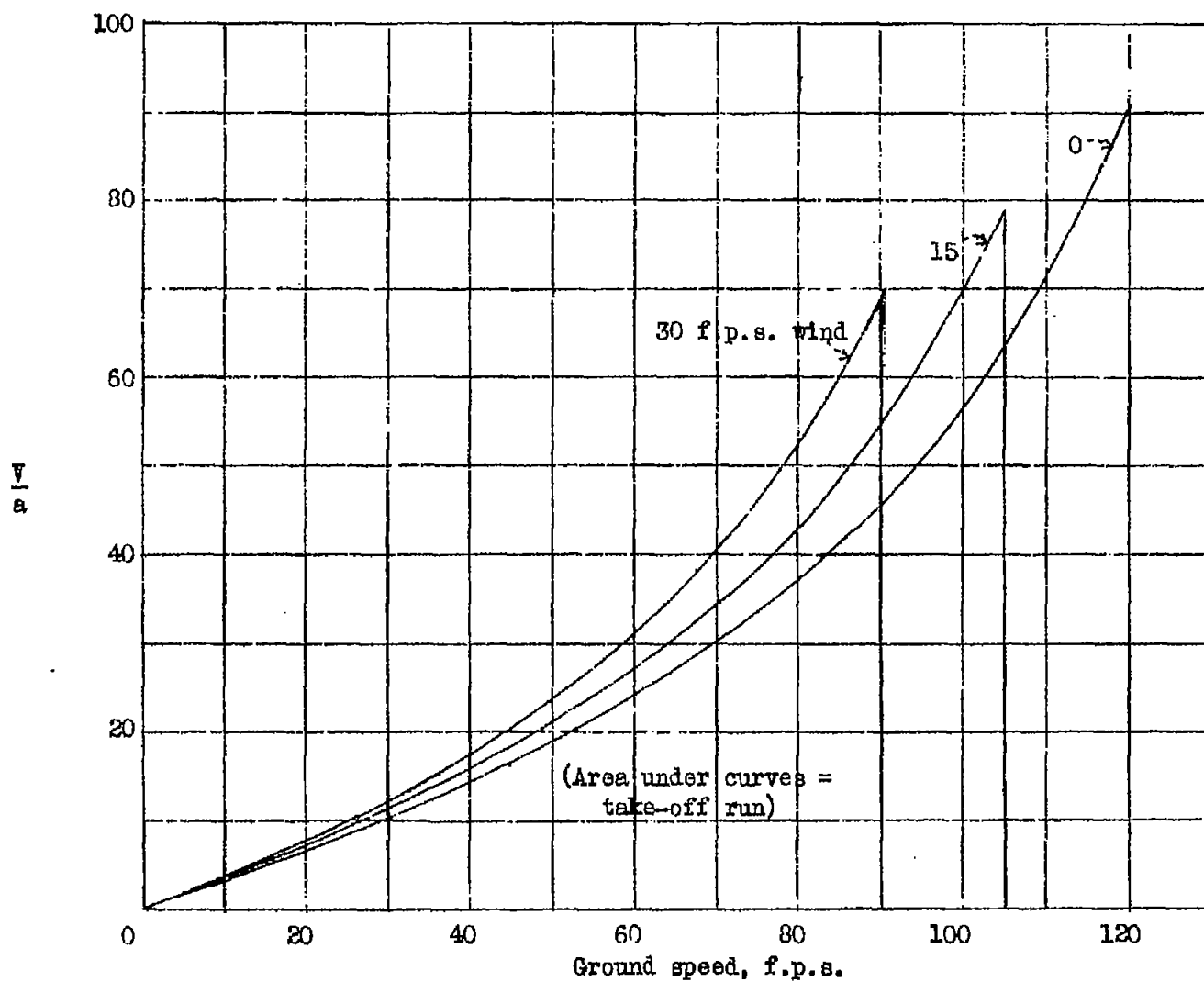


Figure 49.- Diagram illustrating the effect of wind on take-off.
(300 hp. engines, 8.5 ft. diameter propellers.)

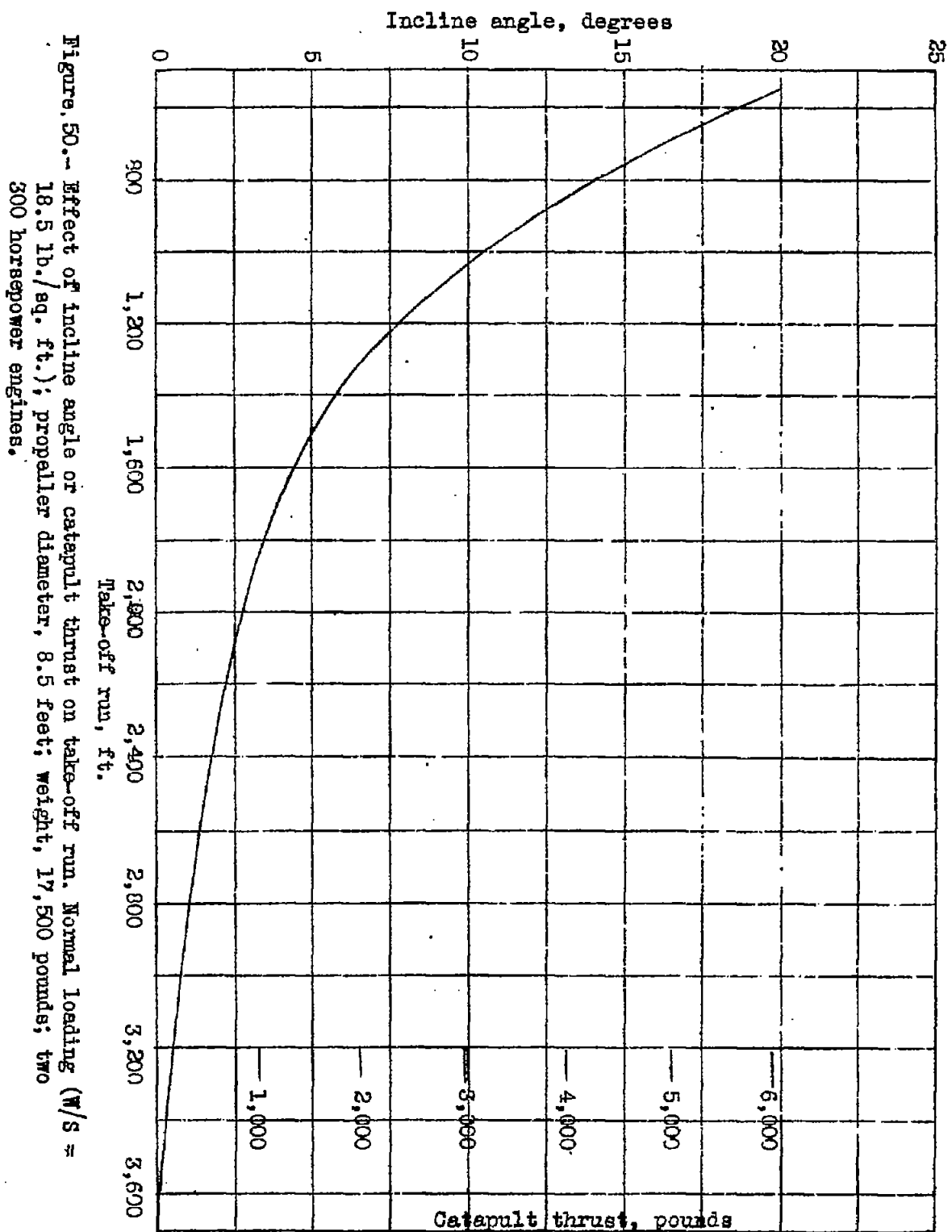


Figure 50.- Effect of incline angle of catapult thrust on take-off run. Normal loading ($W/S = 18.5$ lb./sq. ft.); propeller diameter, 8.5 feet; weight, 17,500 pounds; two 300 horsepower engines.